EX/ FTBE/MATH/T/224/2018

BACHELOR OF ENGINEERING IN FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. EXAM. - 2018

(2ND YR. 2ND SEM.)

MATHEMATICS-III

Time: Three hours Full Marks: 100

GROUP-A(30)

1. (i) $\int_0^{\pi} \frac{x\,dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$

   (ii) $\int \frac{x^4+7x^3+21x^2+33x+20}{x^3+6x^2+11x+6}\,dx$ 6+4

2. The smaller segment of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, cut off by the chord $\frac{x}{a}+\frac{y}{b}=1$ revolves completely about this chord, find the volume of the solid spindle thus generated. 10

3. Find the volume of the solid obtained by the revolution of the cissoid $y^2(2a-x) = x^3$ 10

GROUP-B(30)

4. (a)Solve the following system of equations

$$x+y+z=6$$
$$3x+(3+\epsilon)y+4z=20$$
$$2x+y+3z=13$$

using the Gauss elimination method, where ϵ is small such that $1 \pm \epsilon^2 \cong 1$

(b)Prove that, $\Delta.\nabla = \Delta - \nabla = \nabla.\Delta$ 8+2

5. The following values of the function $f(x)$ for value of x are given: $f(1) = 4, f(2) = 5, f(5) = 6, f(7) = 5$ and $f(8) = 4$. Find the values of $f(4)$ and also the value of x for which $f(x)$ is maximum or minimum. 10

6. Compute by Simpsons one third rule $\int_0^1 (4x-3x^2)\,dx$ by taking $n = 10$, correct to four decimal places and compare the result with the actual value of the integral. Also find absolute and relative errors.

Group-C(10)

7. (a) If $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{pmatrix}$, find the value of $A^3 - 4A^2 - 3A + 11I_3$, hence find A^{-1}.

   (b) Solve the system of equations by Cramer's rule $x+2y-3z=1$, $2x-y+z=4, x+3y=5$ 6+4

Group-D(20)

8. (a) Solve, $k\vec{r} + \vec{r}\times\vec{a} = \vec{b}$, where k is a non-zero scalar and $\vec{a}, \vec{b}$ are two given vectors.

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(b) Given two vectors $\vec{\alpha} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{\beta} = 2\vec{i} - \vec{j} + \vec{k}$; find the vector $\vec{\gamma}$ and the scalar λ which satisfy $\vec{\alpha} \times \vec{\gamma} = \vec{\beta} + \lambda\vec{\alpha}$ and $\vec{\alpha}.\vec{\gamma} = 2$. 5+5

9. (a)Find interm of k, the shortest distance between the lines $\rho = \vec{\alpha} + t\vec{\beta}$ and $\rho = \vec{\gamma} + t\vec{\delta}$, where $\vec{\alpha} = (1,2,3), \vec{\beta} = (2,3,4)$, $\vec{\gamma} = (k,3,4)$ and $\vec{\delta} = (3,4,5)$. For what value of k are the lines coplanar?

   (b)A rigid body is spinning with an angular velocity of 5 radians per second about an axis of direction $(0,3,-1)$ passing through the point $A(1,3,-1)$. Find the velocity of the particle at the point $P(4,-2,1)$. 5+5

Group-E(10)

Answer any one question

10. (a)Show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c}) = -2[\vec{b}\ \vec{c}\ \vec{d}]\vec{a}$

    (b)show that $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$ $7+3$

11. (a)The random variable X has the distribution given by $P(X = k) = 2^{-k}, k = 1,2, \ldots$ find the value of $E(X)$ & $Var(X)$.

    (b)Find the variance for the continues random variable X with probability density function $f(x) = 1 - |1 - x|, 0 < x \leq 2$

    $0, elsewhere$ 10

12. If the probability density function of a random variables X is given by $f(x) = ce^{-(x^2+2x+3)}, -\infty < x < \infty$ find the value of c, the expectation & variance of the distribution.