EX/ FTBE/MATH/T/216/2018(S)

BACHELOR OF ENGINEERING IN FOOD TECHNOLOGY AND
BIO-CHEMICAL ENGG. SUPPLEMENTARY EXAMINATION - 2018

(2^{ND} YR. 1^{ST} SEM.)

MATHEMATICS-II

Time: Three hours Full Marks: 100

GROUP-A

Answer any **five** questions $5\times 2 = 10$

1. (a) State Lagrange's Mean value theorem.
   (b)State Euler's theorem in several variables.
   (c)What is the Demoivre's theorem in complex number?
   (d) If $f(x) = \frac{|x|}{x}$, the derivative exists? Justify your answer
   (e)State Leibnitz's theorem in the n-th derivative of the product of two functions.
   (f)State regular singular point in series solution.
   (g) State Rolle's theorem.

GROUP-B

Answer any **Nine** questions $9 \times 10 = 90$

2. If $y = \cos(m\sin^{-1} x)$, then prove that
   (i) $(1-x^2)y_2 - xy_1 + m^2y = 0$; (ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$ 10
3. If $u = \log r$ *and* $r^2 = x^2 + y^2 + z^2$, then prove that $r^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$ 10
4. If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, then prove that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ and
   (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u$ 10
5. (a)Use Demoivre's theorem to prove, $\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1-10\tan^2\theta + 5\tan^4\theta}$
   (b)Prove that $\cos 5\Theta = 16\cos^5\Theta - 20\cos^3\Theta + 5\cos\Theta$ 10
6. If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate diameters on an ellipse, then find the value of $\rho_1^{\frac{2}{3}} + \rho_2^{\frac{2}{3}}$ 10
7. Represent (x) , where $f(x) = \cos kx$, on $-\pi \le x \le \pi$ (k not being an integer) in Fourier series. Hence deduce that (i) $\pi\cot k\pi = \frac{1}{k} + 2k\sum_{n=1}^{\infty}\frac{1}{k^2-n^2}$
   (ii) $\frac{\pi}{\sin k\pi} = \sum_{n=0}^{\infty}(-1)^n\left\{\frac{1}{n+k} + \frac{1}{n+1-k}\right\}$ 10

[Turn over

8. Find the Fourier series of the function $f(x)$ defined by $f(x) = x + x^2, -\pi \leq x \leq \pi$, hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \dots\dots\dots = \frac{\pi^2}{6}$ 10

9. Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$ 10

10. (a)Solve $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0, where\ D \equiv \frac{d}{dx}$.

    (b)If $\frac{d^2x}{dt^2} = -n^2 x\ and\ x = a, \frac{dx}{dt} = u\ when\ t = 0$, then show that the maximum value of $x\ is\ \sqrt{a^2 + \frac{u^2}{n^2}}$. 5+5

11. Solve the equation $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = x^2$ in series. 10

12. (a)If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(0) + \frac{h^3}{3!} f'''(\theta h), 0 < \theta < 1$ find θ, when $h = 8, f(x) = \frac{1}{1+x}$.

    (b) Find the maximum and minimum values of u where $u = \frac{4}{x} + \frac{36}{y}$ and $x + y = 2$

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