EX/ FTBE/MATH/T/216/2018

BACHOLOR OF ENGINEERING IN FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. EXAMINATION - 2018

**($2^{ND}$ YR. $1^{ST}$ SEM.)**

**MATHEMATICS-II**

Time: Three hours | Full Marks: 100

## GROUP-A

Answer any **five** questions $5\times 2 = 10$

1. (a) State the Dirichlet's condition for the Fourier series.
(b)Explain the periodic function for the Fourier series.
(c) State Euler's theorem in several variables.
(d)State Euler-Cauchy type equation in ordinary differential form.
(e)State Leibnitz's theorem in the n-th derivative of the product of two functions.
(f)State regular singular point in series solution.
(g) State Rolle's theorem.

## GROUP-B

Answer any **Nine** questions $9 \times 10 = 90$

2. (a)If $y = e^{\tan^{-1}x} = a_0 + a_1x + a_2x^2 + a_3 x^3 + \ldots\ldots\ldots\ldots$,find the value of (i) $(1+x^2)y_2 + (2x-1)y_1$ ; (ii) $(1+x^2)y_{n+2} + \{2(n+1)x-1\}y_{n+1} + n(n+1)y_n$ and (iii) $(n+2)a_{n+2} - a_{n+1} + na_n$
(b)Find the n-th derivative of $y = \frac{x^4+7x^3+21x^2+33x+20}{x^3+6x^2+11x+6}$ 7+3

3. (a)If $u = \log(x^3+y^3+z^3-3xyz)$, then find the value of (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$
(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u$ and (iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$
(b) Find the value of $i^i$. 8+2

4. (a)Assuming $f''(x)$ to be continuous in $[a,b]$, show that $f(c) - \frac{b-c}{b-a}f(a) - \frac{c-a}{b-a}f(b) = \frac{1}{2}(c-a)(c-b)f''(\alpha), where\ c\ \&\ \alpha\ both\ lie\ in\ [a,b]$.
(b)Prove that $\sin\left[i\log\frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$ 7+3

5. If $\rho_1$ and $\rho_2$ be the radii of curvature at the extremities of two conjugate diameters on an ellipse, then find the value of $\rho_1^{\frac{2}{3}} + \rho_2^{\frac{2}{3}}$ 10

6. (a) If $u = \sin^{-1}\left\{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$ show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$
(b)Prove that $2^8\sin^9\theta = \sin 9\theta - 9\sin 7\theta + 36\sin 5\theta - 84\sin 3\theta + 126\sin\theta$ 10

7. Represent $(x)$ , where $f(x) = \cos kx$ , on $-\pi \leq x \leq \pi$ (k not being an integer) in Fourier series. Hence deduce that (i) $\pi \cot k\pi = \frac{1}{k} + 2k \sum_{n=1}^{\infty} \frac{1}{k^2-n^2}$

(ii) $\frac{\pi}{\sin k\pi} = \sum_{n=0}^{\infty} (-1)^n \left\{\frac{1}{n+k} + \frac{1}{n+1-k}\right\}$ 10

8. (a)Solve $(D^2 - 4D + 4)y = 8x^3 e^{2x} \sin 2x$ .

(b)Solve $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = 0, where\ D \equiv \frac{d}{dx}$. 7+3

9. Solve the equation $\frac{d^2y}{dx^2} + (x-1)^2 \frac{dy}{dx} - 4(x-1)y = 0$ in series about the ordinary point $x = 1$. 10

10. Let $f(x) = \cos x , for -\pi \leq x \leq 0$

$\sin x \ , for\ 0 < x \leq \pi$

Obtain the Fourier series of $f(x)\ in\ [-\pi, \pi]$ 10

11. (a)If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f'''(\theta h), 0 < \theta < 1$ find $\theta$, when $h = 8, f(x) = \frac{1}{1+x}$.

(b) Find the maximum and minimum values of $u$ where $u = \frac{4}{x} + \frac{36}{y}$ and $x + y = 2$

5+5

12. Solve the equation $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = x^2$ in series. 10