BACHELOR OF ENGINEERING IN FOOD TECHNOLOGY AND
BIO-CHEMICAL ENGG. SUPPLEMENTARY EXAMINATION - 2018
(1ST YR. 1ST SEM.)
MATHEMATICS-I

Time: Three hours   Full Marks: 100

Answer any **Ten** questions   10 × 10

1. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ trans forms to $a'x'^2 + 2h'x'y' + b'y'^2 + 2g'x' + 2f'y' + c'$ under rotation of axes then show that (i) $a' + b' + c' = a + b + c$ (ii) $f'^2 + g'^2 + h'^2 - b'c' - c'a' - a'b' = f^2 + g^2 + h^2 - bc - ca - ab$ and (iii) $2f'g'h' - a'f'^2 - b'g'^2 = 2fgh - af^2 - bg^2$ 10

2. Prove that the transformation of rectangular axes which converts $\frac{x^2}{p} + \frac{y^2}{q}$ into $ax^2 + 2hxy + by^2$ will convert $\frac{x^2}{p-\gamma} + \frac{Y^2}{q-\gamma}$ into $\frac{ax^2+2hxy+by^2-\gamma(ab-h^2)(x^2+y^2)}{1-(a+b)\gamma+(ab-h^2)\gamma^2}$.   10

3. In any conic, prove that

    (i)the sum of the reciprocals of the segments of any focal chord is constant and

    (ii) the sum of the reciprocals of two perpendicular focal chord is constant.   10

4. Find the polar equation of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$, if the pole is at right hand focus and the positive direction of the x-axis is the positive direction of the polar axis.   10

5. If l and l' are the lengths of the segments of any focal chord of the parabola $y^2 = 4ax$, prove that $\frac{1}{l} + \frac{1}{l'} = \frac{1}{a}$.   10

6. Find the equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ and show that the two curves cut one another origin at an angle $\tan^{-1}\frac{3\,a^{\frac{1}{3}}b^{\frac{1}{3}}}{2\left[a^{\frac{2}{3}}+b^{\frac{2}{3}}\right]}$.   10

7. Show that the eccentricity of the ellipse in which the normal at one end of a latus rectum passes through one end of the minor axis is given by the equation $e^4 + e^2 - 1 = 0$ 10

8. A variable plane passes through a fixed point (α, β, γ) and cuts the co-ordinate axes OX, OY and OZ in A, B and C. Show that the locus of the point of intersection of the planes through A, B and C parallel to the coordinate planes is $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 1$.   10

9. Show that the straight lines whose d. cs. are given by $al + bm + cn = 0$, $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$.   10

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10. Show that the equation to the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ and parallel

to the line $\frac{x}{a} + \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$ and if $2d$ is the S.D., Prove that

$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}$.   10

11. A plane passes through a fixed point (α, β, γ) and cuts the co-ordinate axes in A, B, C.

Prove that the locus of the centre of the sphere $OABC$ is given by $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\alpha}{z} = 2$.   10

12. (a)If $y = \left[\log\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)\right]^2 + k\log(x + \sqrt{x^2 - a^2})$ then find the value of $(x^2 - $

$a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = ?$

(b)If $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$, $f'(0) = a, f(0) = b$, then find the value of $f''(x)$,

where y is independent of x.   5+5

13. If $x \cos\theta + y \sin\theta = p$, touch the curve $\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$, then find the value of

$(a\cos\theta)^n + (b\sin\theta)^n = ?$.   10