EX/ FTBE/MATH/T/114/2018

BACHOLOR OF ENGINEERING IN FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. EXAMINATION - 2018

(1^{ST} YR. 1^{ST} SEM.)

MATHEMATICS-I

Time: Three hours | Full Marks: 100

Answer any **Ten** questions | 10×10

1. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ trans forms to $a'x'^2 + 2h'x'y' + b'y'^2 + 2g'x' + 2f'y' + c'$under rotation of axes then show that (i) $a' + b' + c' = a + b + c$ (ii)$f'^2 + g'^2 + h'^2 - b'c' - c'a' - a'b' = f^2 + g^2 + h^2 - bc - ca - ab$ and (iii) $2f'g'h' - a'f'^2 - b'g'^2 = 2fgh - af^2 - bg^2$ 10
2. (a)Prove that the transformation of rectangular axes which converts $\frac{X^2}{p} + \frac{Y^2}{q}$ into $ax^2 + 2hxy + by^2$ will convert $\frac{X^2}{p-\gamma} + \frac{Y^2}{q-\gamma}$ into $\frac{ax^2+2hxy+by^2-\gamma(ab-h^2)(x^2+y^2)}{1-(a+b)\gamma+(ab-h^2)\gamma^2}$.
(b)Show that the foot of the perpendicular from the focus on any tangent to the hyperbola lies on the auxiliary circle. 6+4
3. If the normal is drawn at one extremity of latus rectum PSP' of the conic $\frac{l}{r} = 1 + e\cos\theta$, where S is the pole. Show that the distance from the focus $\frac{l}{r} = 1 + e\cos\theta$,where S is the pole, show that the distance from the focus S of the other point in which normal meets the curve is $\frac{l(1+3e^2+e^4)}{1+e^2-e^4}$. 10
4. If the origin be at one of the limiting points of a system of co-axial circles of which $x^2 + y^2 + 2gx + 2fy + c = 0$ is a member, show that the equation of the system of circles cutting them all orthogonally is $(x^2 + y^2)\,(g + \mu f) + c(x + \mu y) = 0$. Show that the other limiting point is $\left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2}\right)$. 10
5. (a)If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.
(b)Prove that the angle between two straight lines whose d.cs. are given by $l + m + n = 0$and $fmn + gnl + hlm = 0$ is $\frac{\pi}{3}$ if $\frac{1}{f} + \frac{1}{g} + \frac{1}{h} = 0$. 5+5
6. (a)Find the equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ and show that the two curves cut one another origin at an angle $\tan^{-1}\frac{3\,a^{\frac{1}{3}}\,b^{\frac{1}{3}}}{2\left[a^{\frac{2}{3}}+b^{\frac{2}{3}}\right]}$.
(b) Prove that the equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represent a pair of planes, if $c + 2fgh - af^2 - bg^2 - ch^2 = 0$, and also prove that the angle between the planes is $\tan^{-1}\frac{2(f^2+g^2+h^2-bc-ca-ab)^{\frac{1}{2}}}{a+b+c}$ 4+6

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7. Prove that the line of shortest distance between the Z-axis and the variable line $\frac{x}{a}+\frac{z}{c}= \alpha\left(1+\frac{y}{b}\right), \frac{x}{a}-\frac{z}{c}=\frac{1}{\alpha}\left(1-\frac{y}{b}\right)$. 10

8. (a)Show that the locus of a variable line which intersects the three lines $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$ is the surface $y^2 - m^2x^2 = z^2 - c^2$.
(b)Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0, x - 2y + 3z + 1 = 0$ is a great circle. 6+4

9. (a)If $y = \left[\log\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)\right]^2 + klog\left(x + \sqrt{x^2 - a^2}\right)$then find the value of $(x^2 - a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} =?$
(b)If $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}, f'(0) = a, f(0) = b, then\ find\ the\ value\ of\ f''(x)$, where y is independent of x. 5+5

10. (a)If $x\cos\theta + y\sin\theta = p$, touch the curve $\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$, then find the value of $(a\cos\theta)^n + (b\sin\theta)^n =?$
(b)Three normals are drawn from the point $(C, 0)$ to the curve $y^2 = x$, show that C must be greater than $\frac{1}{2}$. One normal is always the x-axis. Find C for which the other two normals are perpendicular to each other. 5+5

11. A line is drawn to meet $y = x\tan\alpha$; $y = -x\tan\alpha$, $z = -c$ so that the length intercepted on it is constant. Show that $\frac{x-k\sin\theta\cot\alpha}{k\cos\theta} = \frac{y-k\cos\theta\tan\alpha}{k\sin\theta} = \frac{z}{c}$ where k is a constant and θ is a parameter. Deduce the equation to the locus of the line. 10

12. A variable plane at a constant distance P from the origin meets the axes at A,B,C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$. 10