Ref. No. : Ex/ET/T/324/2018

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING THIRD YEAR SECOND SEMESTER - 2018

Subject: **DIGITAL CONTROL SYSTEMS** Time: 3 Hours Full Marks: 100

All parts of the same question must be answered at one place only.

1. (a) What is the advantage of analyzing an LTI system using its transfer function? 3

(b) State and prove Nyquist sampling theorem. 7

(c) A sampler cannot be described by a unique transfer function. Justify. 3

(d) A signal $g(t)$ band-limited to B Hz is sampled by a periodic pulse train $p_T(t)$ made up of a rectangular pulse of width $1/8B$ seconds (centered at the origin) repeating at the Nyquist rate ($2B$ pulses per second). Show that the sampled signal is given by 7

$$\bar{g}(t) = \frac{1}{4}g(t) + \sum_{n=1}^{\infty}\left[\frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right)g(t)\cos\left(4\pi nBt\right)\right]$$

Answer Question 2 OR Question 3

2. (a) Describe how the order of a hold circuit determines the extent of smoothening of a sampled signal. 4

(b) Explain the advantage of a fractional order hold circuit over a zero order and a first order hold based on their respective impulse responses. 6

(c) Derive the transfer function of a polygonal hold circuit. 8

(d) Define starred transform of a sampled signal. 2

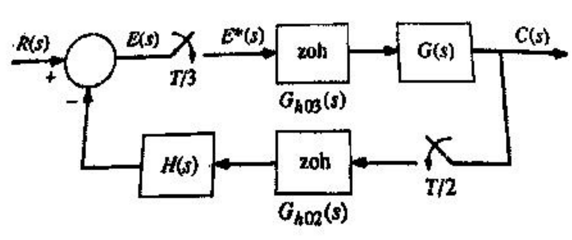
3. (a) Explain how a fast sampler with sampling period T/N can be realized by a slow sampler of sampling period T. $(N>1)$ 5

(b) Determine the output response of a slow-fast sampling system using the model of the fast sampler derived in part (a). 7

(c) How can the same system of part (b) be described using pseudo fast sampler introduced after the slower one? 8

Answer Question 4 OR Question 5

4. (a) Determine the closed loop transfer function for the following multi-rate control system. 10



[Turn over

(b) Given the closed loop transfer function of a digital control system 10

$$\frac{C(z)}{R(z)} = \frac{T^2\left(K_P z^2 + K_I Tz + K_I T - K_P\right)}{Az^3 + Bz^2 + Cz + D}$$

where, $J_v = 41822$, $A = 2J_v$, $B = T^2K_P + 2K_RT - 6J_v$, $C = 6J_v - 4K_RT + T^3K_I$,

$$D = 2K_RT + T^3K_I - 2J_v - T^2K_P$$

Find the values of K_P, K_I and K_R as functions of T so that the step response $c(kT)$ follows the step input in a minimum number of sampling periods. What is the maximum overshoot $c(kT)_{max}$?

5. Consider the following system with the behavior of the plant described by the first order differential equation

$$\frac{d^2y(t)}{dt^2} + 0.15\frac{dy(t)}{dt} + 0.005y(t) = 0.1m(t).$$

(a) Draw a continuous-time simulation diagram for $G_p(s)$ and give the state equations. 7

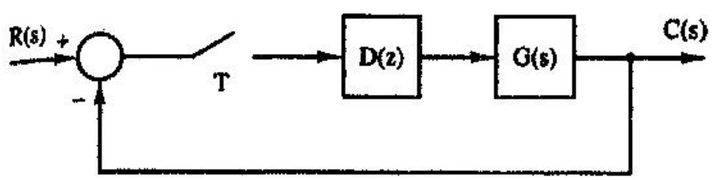
(b) Use the state-variable model of part (b) to find a discrete state model for the entire system. 9

(c) Show that for the similarity transformation 4

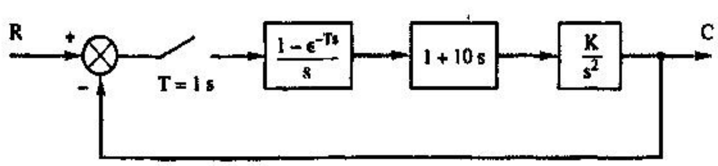
$$\mathbf{C}\left[z\mathbf{I} - \mathbf{A}\right]^{-1}\mathbf{B} + \mathbf{D} = \mathbf{C}_w\left[z\mathbf{I} - \mathbf{A}_w\right]^{-1}\mathbf{B}_w + \mathbf{D}_w.$$

Answer Question 6 OR Question 7

6. (a) Design a digital controller $D(z)$ to attain a steady state error less than 0.01 for unit ramp input and to ensure stability of the entire system with $G(s) = \dfrac{1-\exp(-Ts)}{s(s+1)}$ and T=0.1 sec. 6



(b) Find the range of K for stability of the system from its root locus. 9



(c) Find out the oscillating frequency for the marginal stability of a unity feedback discrete-time control system with $G(z) = \frac{0.368z + 0.264}{z^2 - 1.368z + 0.368} K$. 5

7. (a) State and prove Nyquist stability criterion for digital control system. 12

(b) Using Nyquist stability criteria, comment on stability of a closed loop system with open loop transfer function $\overline{GH}(z) = \frac{0.01kz}{(z-1)(z-0.905)}$. 8

Answer Question 8 OR Question 9

8. (a) For a plant described by 12

$$\vec{x}(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

find the gain matrix K required to realize the closed loop characteristic equation with zeros providing a damping ratio of 0.46 and a time constant of 0.5 s.

(b) Derive the state dynamics of a reduced order state observer. 8

9. (a) Consider a linear digital control system described by 10

$$\vec{x}(k+1) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k).$$

Find the optimal control $u^o(k)$ so that the Lyapunov function $V(\vec{x}) = \vec{x}^T(k)\mathbf{P}\vec{x}(k)$ is minimized where $\mathbf{P}$ is a positive definite solution of $\mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} = -\mathbf{I}$.

(b) Given a first order plant described by $x(k+1) = 0.9x(k) + 0.1u(k)$ with the cost function 10

$$J_3 = \sum_{k=0}^{3} \left(x^2(k) + 5u^2(k) \right)$$

calculate the feedback gains required to minimize the cost function.