10. Solve the following

$$\frac{dx}{dt}+\frac{dy}{dt}-x+5y=t^2$$

$$\frac{dx}{dt}+2\frac{dy}{dt}-2x+4y=2t+1$$ 10

11. Solve the following

a) $\frac{d^2y}{dx^2}+4y=12x^2-16x\cos 2x$

b) $\frac{d^2y}{dx^2}+y=x\sin x.$ 10

12. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\frac{1}{1+x}$ 10

13. Find the solution of

a) $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+3\frac{dy}{dx}-5y=5\sin 2x+10x^2-3x+7.$

b) $\frac{d^2y}{dx^2}+a^2y=\mathrm{Sec}\,ax.$ 10

BACHELOR OF ELECTRONICS & TELE-COMMUNICATION ENGINEERING EXAMINATION, 2018

(1st Year, 2ndSemester)

MATHEMATICS - IV G

Time : Three hours | Full Marks : 100

(50 marks for each group)

Use a separate Answer-Script for each group

GROUP - A

Answer ***any five*** questions

1. a) Use Cramer's rule to solve the system of equation :

$$x+2y=3z+1$$
$$2x+z=x+y$$
$$x+3y=5$$ 5

b) Let S = {(1, 2, 3), (2, 3, 1), (3, 1, 2)}. Then show that S is linearly independent and hence S is a basis of R^3. 5

2. a) Justify with reason whether the following mappings from R^3 to R^3 are linear or not

i) T(z, y, z) = (x + 2y + 3z, y + 2x + 3z, z + 2x + 3y).

ii) S(z, y, z) = (x + 1, 3y, 4z) 5

b) Find the matrix representation of T w.r.t. the basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)} where $T: R^3 \to R^3$ is defined as T(a, b, c) = (a+c, b+a, a + b + c). 5

[Turn over

3. a) Write the following system of equations in the form $AX = B$ and then solve it by finding $A^{-1}B$.

$$\begin{aligned} x + z &= 0 \\ 3x + 4y + 5z &= 2 \\ 2x + 3y + 4z &= 1 \end{aligned}$$ 5

b) Find the rank of the following matrix by reducing it to a row reduced echelon matrix

$$A = \begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$$

4. a) The matrix of a linear mapping $T : R^3 \to R^2$ relative to the ordered bases {(0, 1, 1), (1, 0, 1), (1, 1, 0)} of R^3 and {(1, 0), (1, 1)} of R^2 is

$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$$

Then show that $T(a,b,c) = \left(2a + 2b + c, \frac{1}{2}(-a + b + 3c\right)$,

where $(a,b,c) \in R^3$. 5

GROUP - B

Answer ***any five*** questions

7. Obtain power series solution of

$$\frac{d^2y}{dx^2} - xy = 0$$

in the neighbourhood of $x = 0$. 10

8. Determine the nature of the point x = 0 o the equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2)y = 0.$$

Hence obtain a solution of this equation about the point x = 0 when α is neither zero nor an integer. 10

9. Consider the equation

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -a^2x - 2by.$$

Describe the nature and stability properties of the critical point of the above equation for the following cases

i) $b = 0$ ii) $b = a$ iii) $0 < b < a$. 10

b) Compute AB using block multiplication, where 5

$$A = \begin{pmatrix} 3 & 2 & \cdot & 1 & 0 \\ 0 & 4 & \cdot & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 2 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & \cdot & 2 \\ 0 & 1 & \cdot & 3 \\ \cdot & \cdot & \cdot & \cdot \\ 2 & 3 & \cdot & -1 \\ 1 & -2 & \cdot & 1 \end{pmatrix}$$

b) Find the algebraic and geometric multiplicities of eigenvalues of the following matrix

$$\begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix}$$ 5

5. a) Verify Cayley-Hamilton's theorem for the matrix A, where

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

Hence or otherwise find A^{-1}. 5

b) Find the characteristic polynomial for the matrix

$$A = \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix},$$

where a, b, c are non-zero real numbers. What is the minimal polynomial of the same matrix ? What is the minimal polynomial if $a = c = 0$ but $b \neq 0$. 5

6. a) Let λ be an eigenvalue of a given $n \times n$ matrix A with real entries and $p(x) = a_o + a_1 x + a_2 x^2 + \cdots + a_k x^k$ be any polynomial where the coefficients are real. Then show that $p(\lambda)$ is an eigenvalue of the matrix p(A). Justify whether the converse holds. 5

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$$A = \begin{pmatrix} 3 & 2 & \cdot & 1 & 0 \\ 0 & 4 & \cdot & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 2 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & \cdot & 2 \\ 0 & 1 & \cdot & 3 \\ \cdot & \cdot & \cdot & \cdot \\ 2 & 3 & \cdot & -1 \\ 1 & -2 & \cdot & 1 \end{pmatrix}$$

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