Bachelor of Electronics & Tele-Communication Engineering Examination, 2018

(1st Year, 2ndSemester)

Mathematics - III G

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

Unexplained notations symbols have their usual meanings.

PART - I

1. Justify ***any five*** of the following with proper reasoning.

2×5=10

a) i) $\mathbb{Z}_2 \times \mathbb{Z}_3$ has an element of order 4.

ii) $\mathbb{Z}_2 \times \mathbb{Z}_4$ is not a cyclic group.

b) i) S_3 is a cyclic group.

ii) $P = \{2, 3, 4, \cdots, 10\}$ with divisibility relation is a lattice.

c) i) $x^3 + x^2 + 1$ is an irreducible polynomial in $\mathbb{Z}_2(x)$.

ii) The quotient ring $\mathbb{Z}_2[n]/\left\langle x^3 + x^2 + 1 \right\rangle$ is infinite.

d) For any group G, its set of all subgroups sub(G) can be made into a latticve with $H \vee K = H \bigcup K$ and $H \wedge K = H \bigcap K$.

e) One can define more than one norm on $\mathbb{R}^2$.

[Turn over

f) i) The Boolean polynomial expressing the circuit.

_______ a _______ b _______ is $a + b$.

ii) The projection of $(1, 2, 3) \in \mathbb{R}^3$ on the subspace spanned by $\{(1,0,0), (0,0,1)\}$ is $(1, 2, 0)$.

g) An infinite group can have a proper subgroup with finite number of cosets.

2. Answer ***any two*** : 5×2=10

a) i) Suppose $f : G_1 \to G_2$ is a group homomorphism such that $|G_1| = 10$. Prove that there cannot exist $x \in G_2$ such that $f^{-1}(x)$ has 4 elements.

ii) Give a concrete example (with explanation) of K lien's 4-group 3+2

b) i) find a normal subgroup of S_3.

ii) Give an example of an infinite semigroup having an infinite proper sub semi group which is also a group. 3+2

c) i) Suppose X is a noempty set. Specify the operations so as to make $\rho(X)$ into a ring. Verify if it is a Boolean ring.

ii) Give an example (with explanation) of a field with 4 elements. 2+3

Find the probability of meeting between A and B, if their arrival times are independent and occur at random within the agreed time interval.

9. A letter is known to have come eithe from TATANAGAR or KOLKATA. On the envelop, only two consecutive letter TA are visible. What is the probability that the letter has come from (i) KOLKATA, (ii) TATANAGAR ?

10. Define a Markov chain. What are transient and recurrent state in a Markov chain ? Define irreducibility in a Markov chain and explain when a Markov chain is called irreducible ?

11. Consider a Markov chain on states S={1,2,3,4} and its transition matrix is

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Determine (with explanation) which states are recurrent and which are transient. Check whether the Markov chain is irreducible or not and why ?

d) i) Describe the procedure of making a Boolean ring into a Boolean Algebra and vice-versa.

ii) Every chain is a lattice but not the converse – Explain.

3+3

4. Answer ***any two*** : 6×2=12

a) i) Find an orthogonal basis of $\mathbb{R}^3$ (with the standard inner product) which contains (1, 2, 3).

ii) Can the norm $\|(x_1, x_2)\| = |x_1| + |x_2|$ be induced by an inner product on $\mathbb{R}^2$? Answer with reasons.

iii) Write the formula (no proof is required) for the standard inner product on $\mathbb{C}^2$ ($\mathbb{C}$ denotes the field of complex numbers). 3+2+1

b) i) Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 3 & -2 \end{pmatrix} \in M_{3\times 2}(\mathbb{R})$. Prove that AX = 6

has no solution where $b = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$. Find the best

approximation b such that $AX = \hat{b}$ has a solution. (The inner product under consideration) is assumed to be the standard one). Explain every step properly.

ii) If possible find an inner product <, > on $\mathbb{R}^2$ such that <(1,0), (0,1) $\neq$ 0.

c) i) It is known that Maps (X, F) is a vector space over F with point wise operation where X is a nonempty set & F is a field. Prove that the vector space $M_{2\times 3}(\mathbb{R})$ over $\mathbb{R}$ is of the form Maps(X,F).

ii) It is known that a lattice L need not be distributive, but $a \wedge (b \cup c) \geq (a \wedge b) v (a \cap c)$ is true for all $a, b, c \in L$. Use this to prove that for any three subspaces V_1, V_2, V_3 of a vector space

$V, V_1 \cap (V_2 + V_3) \supseteq (V_1 \cap V_2) + (V_1 \cap V_3)$.

PART - II

Answer ***any five*** questions ×5=50

5. A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the distribution of the number of success. Also find the mean and the variance of the number of successes.

6. There are three identical urns containing white and black balls. The first urn contains 2 white and 3 black balls, the second urn 3 white and 5 black balls and the third urn 5 white and 2 black balls. An urn is chosen at random and a ball is drawn from it. If the ball drawn is white, what is the probability that the second urn is chosen ?

7. In a normal distribution, 10.03% of the items are under 25 kilograms weight and 89.97% of the items are under 70 kilograms weight. What are the mean and the standard deviation of the distribution ?

   (Given that $P(0 < Z < 1\cdot 28) = 0\cdot 3997,$ where Z is the standard normal variate). Draw a rough normal curve indicating the appropriate point.

8. Two persons A and B agree to meet at a given place between 12 noon and 1 p.m. with the understanding that the first to arrive shall wait for 20 minutes for the other and then leave.

3. Answer ***any two*** : 6×3=18

   a) Does there exist a nondistributive lattice with (i) 6 elements, (ii) infinite number of elements ? Justify your answer in each case.

   b) Let P = [0, 1]. Define $\vee, \wedge, '$ on P as follows :

   $$a \vee b := \max\{a, b\}, a \wedge b = \min\{a, b\},$$

   $$a' = 1 - a \text{ for all } a, b \in P.$$

   Prove that it is a bounded distributive lattice and that it is not a Boolean Algebra.

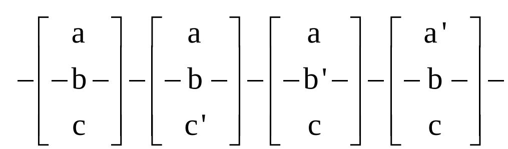
   Are De Morgan's laws true here ? Justify your answer.

   2+4

   c) i) A committee of three persons A, B, C decide any proposal by a majority of votes.

   A has voting weight 3, B has voting weight 2 and C has voting weight 1. Design a simple circuit by the Boolean Algebraic technique so that the light will glow when a majority of votes is cast in favour of a proposal.

   ii) Simplify the following switching circuit.



3+3