B.E.T.C.E. Supply. Examination, 2018(S)

(1st YR, 1st SEM)

MATHEMATICS

PAPER - II G

Full Marks : 100 **Time: Three hours**

Answer any question 1 and any six from the rest. $4 + 6 \times 16 = 100$

1. Prove that

$$\lim_{z \to 0} = \frac{\overline{z}}{z}$$ 4

does not exist.

2. (a) Show that the necessary and sufficient condition for a vector function $\overrightarrow{F}(t)$ to have constant direction is

$$\overrightarrow{F}(t) \times \frac{d\overrightarrow{F}(t)}{dt} = 0$$

(b) Find the directional derivative of a scalar point function $f(x, y, z)$ along any line whose direction cosines are l, m, n. 8+8

3. (a) Define with examples of regular point, singular point, isolated singularity and removal singularity.

(b) Evaluate the residues of $f(z)$ where

$$f(z) = \frac{e^2}{z^2(z^2+9)} \quad at\ z = 0.$$

8+8

4. (a) Define Harmonic function. Show that $u(x,y) = 2x-x^3+3xy^2$ is harmonic and find its harmonic conjugate and corresponding analytic function.

(b) Prove that the real and imaginary parts of an analytic function are harmonic. 8+8

5. (a) Find the constants a, b so that the surfaces

$$ax^2 - byz = (a+2)x$$

will be orthogonal to the surface

$$4x^2y + z^3 = 4, \quad at \quad (1, -1, 2).$$

(b) State Gauss Divergence theorem.
Verify Gauss Divergence theorem for

$$\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$$

taken over the region bounded by

$$x^2 + y^2 = 4, \quad z = 0 \quad and \quad z = 3.$$

8+8

6. (a) Find the analytic function $f(z) = u + iv$ of which the complex part is

$$v = 6xy - 5x + 3.$$

(b) Find the value of the

$$\oint_C \frac{dz}{1+z^2},$$

where C is the contour

$$\left|z - \frac{i}{2}\right| = 1.$$

8+8

7. (a) If $\vec{A}$ and $\vec{B}$ are irrotational , then prove that $\vec{A} \times \vec{B}$ are solenoidal.

(b) Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative field and find a function ϕ such that $\vec{\nabla}\phi = \vec{F}$. 8+8

8. (a) If

$$\vec{f} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k},$$

evaluate

$$\int_S (\vec{\nabla} \times \vec{f}).\vec{n}\, dS,$$

where, S is the surface

$$x^2 + y^2 + z^2 = a^2,$$

above xy plane.

(b) Show that

$$curl\ curl\ curl\ curl F = \nabla^4 \vec{F}\ where\ div \vec{F} = 0.$$

(8+8)