Ex/ET/MATH/T/114/2018

B.E.T.C.E. Examination, 2018
(1ST YR, 1ST SEM)

MATHEMATICS - IIG

Full Marks : 100 **Time: Three hours**

Answer any question 1 and any six from the rest. $4 + 6 \times 16 = 100$

1. Show that the following function is not differentiable but C-R equation satisfied at z=0

$$f(z) = \begin{cases} \frac{xy}{x^2+y^2}, if z \neq 0 \\ 0, if z = 0 \end{cases}$$

2. (a) Define Harmonic function. Show that u(x,y) = 2x-x^3+3xy^2 is harmonic and find its harmonic conjugate and corresponding analytic function.

(b) Show that an analytic function in a region with constant modulus is constant.

(c) Prove that the real and imaginary parts of an analytic function are harmonic.

8+4+4

3. (a) Evaluate

$$\int_C f(z)dz,$$

where

$$f(z) = y - x - i3x^2,$$

and C is the line segment from z=0 to z=1+i.

[Turn over

(b) Evaluate using Cauchy's integral formula.

(i)

$$\int_C \frac{sin^2 z}{(z-\frac{\pi}{6})^3} dz,$$

where C is the unit circle.

(ii)

$$\int_C \frac{e^z + zsinhz}{(z-i\pi)^2} dz,$$

where C is the circle $|z| = 4$. (6+5+5)

4. (a) Define singularity of a function f(z). Determine and classify the singular points of the following functions

$$(i)\ ze^{\frac{1}{z}} \qquad (ii)\ \frac{1}{z(z-1)^2} \qquad (iii)\ \frac{z}{e^z - 1}$$

(b) What is the residue of a function f(z)?

(c) Calculate the residue of the following functions at the poles

$$(i)\ \frac{z+1}{z^2-2z} \qquad (ii)\ \frac{e^z}{z^2(z^2+9)}$$ $(8+2+6)$

5. Evaluate using Cauchy's Residue theorem

(i)

$$\int_C \frac{z^2}{(z-2)(z+3)} dz,$$

where C is the circle $|z| = 4$

(ii)

$$\int_C \frac{3z-4}{z(z-1)} dz,$$

where C is the circle $|z| = 2$ (8+8)

6. (a) Show that the necessary and sufficient condition for a vector function $\vec{F}(t)$ to have constant direction is

$$\vec{F}(t) \times \frac{d\vec{F}(t)}{dt} = 0$$

(b) Find the directional derivative of a scalar point function $f(x, y, z)$ along any line whose direction cosines are l, m, n. (8+8)

7. (a) Find the angle between two surfaces

$$xy^2z = 3x + z^2 \quad and \quad 3x^2 - y^2 + 2z = 1 \quad at \ (1, -2, 1).$$

(b) Evaluate

$$\int_C \vec{F} \cdot d\vec{r},$$

where

$$\vec{F} = (x^2 + y^2)\vec{i} - 2xy\,\vec{j}$$

and the curve C is the rectangle in the xy plane bounded by

$$y = 0, \ x = a, \ y = b, \ x = 0.$$

(c) If $\nabla \times \vec{A} = 0$ and $\nabla \times \vec{B} = 0$, then prove that $\nabla \cdot (\vec{A} \times \vec{B}) = 0$.

(5+7+4)

8. (a) If

$$\vec{f} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k},$$

evaluate

$$\int_S (\vec{\nabla} \times \vec{f}).\vec{n}\, dS,$$

where, S is the surface

$$x^2 + y^2 + z^2 = a^2,$$

above xy plane.

(b) Show that

$$curl\ curl\ curl\ curl F = \nabla^4 \vec{F} \ where \ div \vec{F} = 0.$$

(8+8)