B. ETCE 1st year 1st sem Suppl. 2018

Mathematics-I G

Use seperate answer sheet for each group

Group-A

(Answer any five questions)

Q.1.(i) If $y = e^{(mcos^{-1}x)}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ and find $y_n(0)$. **5**

Q.1.(ii) State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$. **5**

Q.2.(i) Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0, \forall x \in (0,1)$. Find the intervals in which $g(x)$ is increasing and decreasing. **5**

Q.2.(ii) Show that $\frac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. **5**

Q.3.(i) Examine the extreme value, if $f(x) = x^2(x-1)^3$. **5**

Q.3.(ii) Show that of all rectangles of given area, the square has the smallest perimeter. **5**

Q.4.(i) Determine $\lim\limits_{x\to 0} \frac{1+\sin x - \cos x + \log(1-x)}{x\tan^2 x}$. **5**

Q.4.(ii) Expand $\log_e(1+x)$ in a finite series in powers of x, with remainder in Lagrange's form. **5**

Q.5. Find all the asymptotes of the curve $xy^2 - x^2y = a^2(x+y) + b^2$. **10**

Q.6.(i) Prove that the sequence $\{x_n\}$ defined by

$$x_n = \frac{3n-1}{n+2}$$

is convergent. **5**

Q.6.(ii) Examine the convergence of the series **5**

$$\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \frac{4}{7.9} + \dots$$

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Q.7.(i) Let $f_n(x) = x^2 e^{-nx}, x \in [0, \infty)$. Show that $f_n(x)$ is uniformly convergent on $[0, \infty)$. **5**

Q.7.(ii) Prove that the series $\sum \frac{\cos nx}{n(n+1)}$ is uniformly convergent for all real x. **5**

Group-B

(Answer any five questions)

Use seperate answer sheet for each group

Q.8. Evaluate $\lim_{x\to 0} \lim_{y\to 0} f(x,y)$, $\lim_{y\to 0} \lim_{x\to 0} f(x,y)$ and $\lim_{(x,y)\to(0,0)} f(x,y)$, if they exist, for the following functions: **5+5**

$$\begin{aligned}(i)\ \ f(x,y) &= \frac{x^3+y^3}{x-y}\ , \quad x \neq y \\ &= 0 \qquad\quad , \quad x = y \\ (ii)\ \ f(x,y) &= \frac{xy}{x^2+y^2} + y\sin\frac{1}{x}\ , \quad xy \neq 0 \\ &= 0\ , \qquad\qquad\qquad x = 0,\ y = 0\end{aligned}$$

Q.9.(i). Show that the function $f : R^2 \longrightarrow R$ defined by

$$\begin{aligned}f(x,y) &= \frac{x^2+3y^2}{3x^2+y^2}, (x,y) \neq (0,0) \\ &= 0, (x,y) = (0,0)\end{aligned}$$

is not continuous at the origin(0,0). **5**

Q.9.(ii). If $u = \tan^{-1} \frac{x^3+y^3}{x-y}$ then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} - sin2u = 0$$

5

Q.10.(i). Let $f : R^2 \longrightarrow R$ defined as

$$\begin{aligned} f(x,y) &= (x^2+y^2) sin\frac{1}{\sqrt{x^2+y^2}} \;, \quad (x,y) \neq (0,0) \\ &= 0 \;, \qquad (x,y) = (0,0) \end{aligned}$$

Then show that f is differentiable at (0,0). Verify whether f_x or f_y is continuous at (0,0) or not. **6**

Q.10.(ii). Find the directional derivative of the function $f(x,y,z) = \log(x^2+2y^2+z^2)$ at the point $(2,1,1)$ in the direction of $(-1,2,3)$. **4**

Q.11.(i) State Euler's Theorem and use it to show that if $\sin u = \frac{x^2+y^2}{x+y}$ then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$

5

Q.11.(ii) Find the equation of the tangent plane to the surface defined by $f(x,y) = 2\cos(x-y) + 3\sin x$ at the point $(\pi, \pi/2)$. **5**

Q.12. (i) If $x = r\cos\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$ then show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta.$$

5

Q.12.(ii) State Weierstrass's M-test. Use it to test the uniform convergence of the series $\sum f_n(x)$, where $f_n(x) = \frac{\sin nx}{x^2+n^2}, x \geq 1$. **5**

Q.13. (i) Find and classify the extreme values (if any) of the function defined as : **5**

$$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$

Q.13.(ii). Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$. **5**

Q.14. Find the minimum value of $x^2+y^2+z^2$ subject to the condition $ax+by+cz = k^2$. **10**