Ref No: Ex/EE/5/T/421/2018

B.E. ELECTRICAL ENGINEERING (PART TIME) FOURTH YEAR SECOND SEMESTER EXAMINATION 2018

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Full Marks 100

Time: Three hours **(50 marks for each part)**

Use a separate Answer-Script for each part

<table>
<tr><th>No. of Questions</th><th>PART- I</th><th>Marks</th></tr>
<tr><td></td><td>Answer any <i>THREE</i> questions
Two marks reserved for neatness and well organized answers.</td><td></td></tr>
<tr><td>1. (a)</td><td>It is claimed that the number (<i>Y</i>) of messages sent per hour over a digital communication system, has the following probability distribution:
<table>
<tr><td>y=Number of Messages</td><td>25</td><td>35</td><td>45</td><td>55</td><td>65</td><td>75</td></tr>
<tr><td>$W(y)$</td><td>0.32</td><td>0.25</td><td>0.15</td><td>0.12</td><td>0.10</td><td>0.06</td></tr>
</table>
Check whether or not <i>W(y)</i> is a valid probability mass function (PMF).
Determine the mean, the mean-square value and the variance of the number of messages sent per hour.</td><td>8</td></tr>
<tr><td>(b)</td><td>State clearly the properties of probability density function (pdf) of continuous random variables.</td><td>8</td></tr>
<tr><td>2. (a)</td><td>Cite with explanations, three practical applications of uniform probability distribution .</td><td>9</td></tr>
<tr><td>(b)</td><td>Define “Moment Generating Function” (MGF). Derive the expression for the MGF of uniformly distributed random variable. From that expression, obtain the expressions for the mean value and the variance of the random variable.</td><td>7</td></tr>
<tr><td>3. (a)</td><td>What do you mean by statistical independence of two continuous random variables <i>X</i> and <i>Y</i> ? In such a case, how is the joint probability density function (pdf) <i>f(x,y)</i> related to the marginal density functions <i>f(x)</i> and <i>f(y)?</i> Explain.
Show that if two random variables <i>X</i> and <i>Y</i> are s-independent, they are also statistically uncorrelated.</td><td></td></tr>
</table>

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No. of Questions	**PART I**	**Marks**
	If $X_1, X_2, X_3,......, X_N$ are s-independent random variables, and if $Y= X_1+ X_2+ X_3+......+X_N$, how is the pdf of Y related to those of $X_1, X_2, X_3,......, X_N$? Give derivations.	**2+4+4**
(b)	**Suppose that an amplifier of a certain type, when subjected to an accelerated life test, has a lifetime X (in weeks) that has an exponential distribution with a mean of 20 weeks** **(i) What is the probability that the amplifier will survive up to 10 weeks?** **(ii) What is the probability that the amplifier will fail before 12 weeks?** **(iii) What is the standard deviation of the life of the amplifier?** **Derive the expressions used.**	**6**
4. (a)	**The line width for semiconductor manufacturing is assumed to have a Gaussian distribution with a mean of 0.5 µm and a standard deviation of 0.05 µm.** **(i) What is the probability that a line width is greater than 0.62 µm ?** **(ii) What is the probability that a line width is between 0.47 and 0.65 µm?** **(iii) What is the value W(µm), such that the line width of 90% of samples is below W?** **Use the attached table for the cumulative distribution function of standard normal (Gaussian) random variable.**	**10**
(b)	**If a random variable X has a mean µ and a standard deviation σ, Determine the expression for the random variable $Y= aX+b$, (where a and b are constants) in terms of µ and σ.**	**6**

	PART- I	
5.	**Write short notes on <u>any two</u> of the following.**	
(a)	**Characteristic function of random variables and its application.**	**8+8**
(b)	**Binomial distribution.**	
(c)	**Poisson distribution.**	
(d)	**Coefficients of ‘Skewness’ and ‘Kurtosis’.**	
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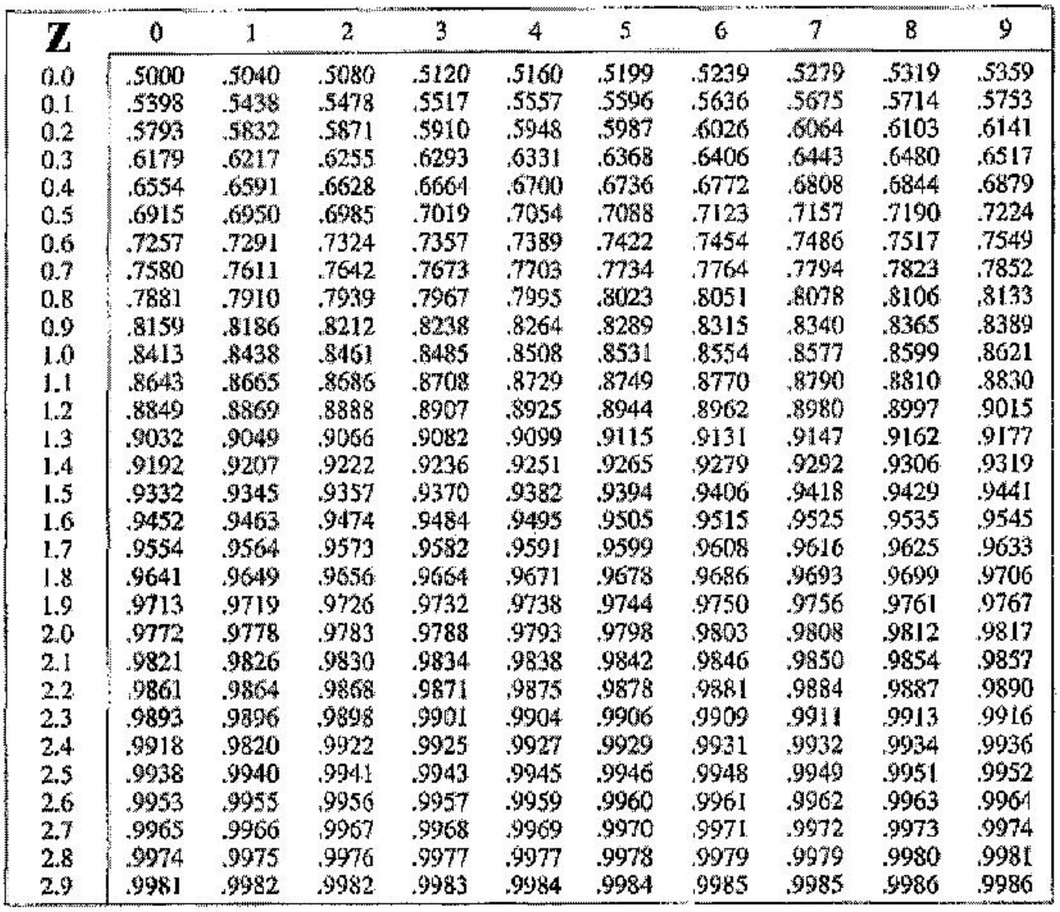
Table. Standard normal distribution

F(z) for z= 0.00– 2.99



Z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9820	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

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BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING (EVENING)EXAMINATION, 2018

(4th Year, 2nd Semester)

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHOD

Time: Three Hours Full Marks: 100

(50 marks for each part)

Use a separate Answer-script for each Part

PART-II

Answer any *three questions*

(Two marks are reserve for neatness and well organized answers)

1.a) A and B are the least squares estimators of α and β in a linear regression model and the random errors are independent normal random variables having mean '0' and variance σ2. Establish that A and B are unbiased estimators of α and β. 8

b) Discuss the technical inferences concerning α and β. 8

2. a) Establish the relationship between sample variance and population variance. 4

b) State and discuss the central limit theorem. 4

c) The amount of water used per day by a person residing in a city has a mean value of 147 liters with a standard deviation of 62 liters. If a random sample of 25 persons is taken, approximate the probability that the average amount of water consumed by member of this group exceeded 150 liters. 8

3. a) Explain the hypothesis test concerning the mean of a normal population. 8

b) Describe significance levels associated with hypothesis testing. 8

4. a) Suppose X1, ….. , Xn are independent Poisson random variables each having mean λ. Determine the maximum likelihood estimator of λ. 8

b) A signal having value μ is transmitted from location A and the value received at location B is normally distributed with mean μ and variance 4. If the successive values received are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, and 10.5. Find a 95% confidence interval for μ. 8

[Turn over

5. Write short notes on any two of the following:2x8=16

a) The t – distribution

b) The Bayes Estimators

d) The Chi – Square distribution

For solving the numerical problems the standard normal probability distribution function table provided below may be used with linear interpolation/extrapolation

z	0.1	0.2	0.3	1.8	1.9	2.0
$\phi(z)$	0.5398	0.5793	0.6179	0.9641	0.9713	0.9772

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