Ref. No.: EX/EE/5/MATH/T/113/2018

BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING (EVENING) EXAMINATION, 2018

(1st Year, 1st Semester)

Mathematics-IIF

Time: Three hours

Full Marks: 100

(Symbols and notations have their usual meanings)
Answer any *five* questions

1. a) Prove by vector method $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ in a triangle ABC. 5

   b) Find a unit normal to the surface $2x^2y + 3yz = 4$ at the point (1, -1, -2). 5

   c) Show by vector method that the medians of a triangle are concurrent. 5

   d) Show that $\vec{\nabla}(\vec{F}.\vec{G}) = \vec{F} \times (\vec{\nabla} \times \vec{G}) + \vec{G} \times (\vec{\nabla} \times \vec{F}) + (\vec{G}.\vec{\nabla})\vec{F} + (\vec{F}.\vec{\nabla})\vec{G}$. 5

2. a) If $\vec{F} = 3xy\,\hat{\imath} - 5z\,\hat{\jmath} + 10x\,\hat{k}$, then evaluate $\int_C \vec{F}.d\vec{r}$ along the curve C given by $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. 5

   b) Show that the vector $\vec{F} = (4xy - z^3)\hat{\imath} + 2x^2\hat{\jmath} - 3xz^2\hat{k}$ is a conservative force field. Show that $\vec{F}$ can be expressed as the gradient of some scalar point function φ. 7

   c) Verify Stoke's theorem for the vector function $\vec{F} = (x^2 - y^2)\hat{\imath} + 2x\hat{\jmath}$ around the rectangle bounded by straight lines x=0, x=a, y=0, y=b. 8

3. a) Evaluate $\iint_S \vec{F}.\hat{n}\, dS$ where $\vec{F} = 3xz\,\hat{\imath} + y^2\,\hat{\jmath} - 3yz\,\hat{k}$ and S is the surface of the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$. 7

   b) Show that the vector $\vec{F} = \sin y\,\hat{\imath} + \sin x\,\hat{\jmath} + e^{10z}\hat{k}$ is not irrotational. 3

   c) Solve the following Euler-Cauchy differential equation 10
   $$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x^2.$$

4. a) Solve the following: 2x5=10

   i. $(x^3 - 3x^2y + 2xy^2)\,dx - (x^3 - 2x^2y + y^3)dy = 0$

   ii. $(e^x x^6 + 4y)dx - x\,dy = 0$

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b) Solve by the method of variation of parameters 10

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}.$$

5. Solve: 4x5=20

a) $x\,dx + y\,dy + \frac{xdy - ydx}{x^2+y^2} = 0$

b) $(4x^2y - 6)\,dx + x^3dy = 0$

c) $(D^2 - 5D + 6)y = 8e^{-x}$

d) $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)\,dy = 0$

6. a) Solve the equations by Cramer's rule,

$$2x - z = 1,$$
$$2x + 4y - z = 1,$$
$$x - 8y - 3z = -2.$$

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b) Show that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (b+a)^2 \end{vmatrix} = 2abc(a+b+c)^2.$ 7

c) Find the rank of the matrix $\begin{pmatrix} 2 & 3 & -1 & 1 \\ 3 & 0 & 4 & 2 \\ 6 & 9 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 7

7. a) Verify Cayley-Hamilton theorem for the matrix

$A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$. 6

Hence obtain A^{-1}.

b) Find the eigen values and the corresponding eigen vectors of the matrix

$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ 9

c) If A, B be two symmetric matrices of the same order, then show that $A + B$ is symmetric iff $AB = BA$ 5

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