d) A die is thrown 108 times in succession. Find the expectation and variance of the number of ‘six’ appeared. 6

8. a) Find the mean and variance of Binomial distribution. 8

b) An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is divisible by 6 or 8? 6

c) For any events A and B, show that

$P(A \cap B) \leq P(A) + P(B)$. 6

Bachelor of Engineering in Electrical Engineering (Evening) Examination, 2018

(1st Year, 1st Semester, **Old Syllabus**)

Mathematics - IVF

Time : Three hours Full Marks : 100

(Symbols and notations have their usual meanings)

Answer ***any five*** questions.

1. a) Find a unit normal vector to the surface $2x^2y + 3yz = 4$ at the point $(1, -1, -2)$. 4

b) Show that

$\vec{\nabla} \times (\vec{F} \times \vec{G}) = \vec{F}(\vec{\nabla} \cdot \vec{G}) - \vec{G}(\vec{\nabla} \cdot \vec{F}) + (\vec{G} \cdot \vec{\nabla})\vec{F} - (\vec{F} \cdot \vec{\nabla})\vec{G}$. 6

c) Find the maximum value of the directional derivatives of the function $f = x^2 + y^2 - z^2$ at the point $(1, 3, 2)$. Find also the direction in which it occurs. 7

d) Show that the vector $\vec{F} = \sin y\,\hat{i} + \sin xy^2\,\hat{j} + e^z\hat{k}$ is not solenoidal.

2. a) Show that the vector $\vec{F} = x^2\hat{i} + y^2\,\hat{j} + z^2\hat{k}$ can be expressed as the gradient of some scalar point function φ. 5

b) Verify Green’s theorem in plane for $\oint_C [(3x^2 - 6y^2)dx + (y - 3xy)dy]$ where C is the boundary of the region x=0, y = 0, x + y = 1. 7

[Turn over

c) Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where $\vec{F}=2x^2\hat{i}+y\hat{j}-z^2\hat{k}$ and S is the surface of the cylinder $x^2+y^2=16$ between the planes $z=0$ and $z=2$ together with the circular ends of these planes. 8

3. a) Show that $f(z)=\begin{cases}\dfrac{xy^2(x+iy)}{x^2+y^4} & \text{if } z\neq 0\\ 0 & \text{if } z=0\end{cases}$

b) Evaluate $\int_C \frac{z^2}{(z-2)(z+3)}dz$ where C is the circle $|z|=4$. 7

c) Find the Laurent's series expansion of the function $\frac{1}{z^2-3z+2}$ valid in the region $1<|z|<2$. 6

4. a) Find the directional derivative of $\phi = xy^2z + 4x^2z$ at (–2, 1, 2) in the direction $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$. 5

b) Find the equations of the tangent plane and normal line to the surface $xyz=4$ at the point $\hat{i}+2\hat{j}+2\hat{k}$. 8

c) Show that the vector

$$\vec{F}=\left(y^2+z^3\right)\hat{i}+(2xy-5z)\hat{j}+\left(3xz^2-5y\right)\hat{k} \text{ is irrotational.}$$

Show that $\vec{F}$ can be expressed as the gradient of some scalar function ϕ. 7

5. a) Using the method of contour integration evaluate

$$\int_{-\infty}^{\infty}\frac{x^2}{\left(x^2+4\right)\left(x^2+1\right)}dx.$$ 8

b) Evaluate $\int_C \frac{zdz}{z^2-1}$ where C is the positively oriented circle $|z|=2$. 7

c) Verify Cauchy - Riemann equations for the function $f(z)=z^3$. 5

6. a) State and prove Bay's theorem. 6

b) Find mean and variance of Normal distribution. 8

c) An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is divisible by 6 or 8 ? 6

7. a) Define : Random experiment, certain event. 3

b) For any two events A and B, show that

$$P(A)=P(A\cap B)+P\left(A\cap\overline{B}\right).$$ 4

c) Show the function f(x) given by

$$f(x)=\begin{cases}|x|, & -1<x<1\\ 0 & \text{elsewhere}\end{cases}$$

is a probability density function. Also find corresponding probability distribution function. 7