Ref. No. : Ex/EE/T/423A/2018

B.E. ELECTRICAL ENGINEERING FOURTH YEAR SECOND SEMESTER EXAM 2018
NONLINEAR AND OPTIMAL CONTROL (SPECIAL PAPER-I)

(50 marks for each Part)

Time : 3 hours | Use a separate Answer-Script for each Part | Full Marks : 100

Part-I

Answer any THREE questions.

Different parts of the same question should be answered together. Two marks will be given for neat and well organized answer.

1. a) What is the objective of optimal control theory?

b) State the advantages and disadvantages of optimal control.

c) Explain how selection of proper mathematical model may influence the formulation optimal control problem.

[2+4+10=16]

2. a) Define the following:

(i) Control history, (ii) State trajectory, (iii) Admissible control, (iv) Admissible trajectory

(v) Function (vi) Functional

b) A car is parked at some point O and it is to be driven to point e in a straight line. The distance of the car from O at a given time is denoted by d(t) . For simplicity, the car may be considered as a point mass that can be accelerated by using a throttle and decelerated by a brake. The equation of motion of the car may be expressed as:

$$\ddot{d}(t) = \alpha(t) + \beta(t)$$

Where α is the throttle acceleration and β is braking deceleration. The maximum acceleration is $M_1 > 0$, and maximum deceleration is $M_2 > 0$.

Formulate a suitable optimal control problem for admissible states and control.

[6+10=16]

3.a) What is performance measure and what is its role in optimal control problem?

b) Explain different types of performance measures commonly selected for the formulation of optimal control problems. [2+2+12=16]

4. a) Explain the following with example:

(i) Closeness of Functions, (ii) Increment of Functional (iii) Variation of a Functional

[Turn over

b) Let x be a continuous scalar function defined for t ϵ [0, 1]. Find the variation of the functional:

$$J(x) = \int_0^1 [x^2(t) + 2x(t)]dt$$

[3x3+7=16]

5. a) (i) Explain the principal of formulation of a linear state regulator problem for a **SISO** LTI system.

(ii) Explain the Steepest Descent Method for finding solution of a two point boundary value problem.

[8+8=16]

OR

b) Explain the computational procedure of solving optimal control problem by Dynamic Programming.

[16]

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B.E. ELECTRICAL ENGINEERING FOURTH YEAR SECOND SEMESTER EXAMINATION, 2018

Subject: Nonlinear and Optimal Control

Full Marks 100

Time: Three hours **Part-II** **(50 marks for each part)**✓

Use a separate Answer-Script for each part

Answer all the questions.

6a.) Consider a mass-spring-damper system with a position dependent damping coefficient. Represent the dynamics of the system with a second order nonlinear differential equation. What is the typical name of this equation?

Using the basic knowledge of physics explain, why such a system does exhibit limit cycle oscillation.
Draw suitable plots, for two different initial conditions, which depict the limit cycle oscillations.

8

OR

6b.) Consider a nonlinear system whose dynamics can be represented by a Duffing equation. State the condition under which the system will exhibit the phenomenon of Bifurcation.
What do you understand by the terms "Critical or Bifurcation values"?
In this context, explain the phenomenon of Pitchfork Bifurcation.
Draw a suitable diagram with the help of which the nonlinear behavior of the system could be understood. Derive the equilibrium points and explain their significances with the help of the plot.

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7a.) Consider a linearized second-order autonomous system described by the dynamics, (d**X**/dt) = A**X** andhaving eigenvalues which are both real and distinct. Find out the equilibrium point/s of the system and thereby predict the stability of the equilibrium point/s. Draw the corresponding Phase Portraits in x_1-x_2 coordinate system.

Next, consider a different set of state variables defined as **X**= **MZ**; **M** is a modal matrix.

Prove that the phase portraits under this transformation will be a set of parabolas in z_1-z_2 coordinate system. Draw the Phase portraits in transformed coordinates. **12**

OR

7b.) Calculate the slope gradient and draw an approximate phase portrait for the following system using isoclines method

$$\ddot{\theta} + \dot{\theta} + 0.5\theta = 1$$

12

8a.) Obtain the describing function for backlash nonlinearity. Plot the describing function. **12**

OR

8b.) What is the basic concept of feedback linearization technique? Consider a system described by the following state equations. Obtain a suitable feedback linearized model of the system. Also find the expression for the control input as a function of the transformed state variables that would lead to a feedback linearized system.

$$\dot{x}_1 = -2x_1 + ax_2 + \sin x_1 \quad and$$
$$\dot{x}_2 = -x_2 \cos x_1 + u\cos(2x_1)$$

12

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9a.) Write short note on Jacobian linearization technique and with the help of a suitable example that employ the technique for obtaining a locally linearized model of a given nonlinear system explain the technique. **6**

OR

9b.) State and explain the theorem of Lyaupunov's linearization method which relates the stability of the equilibrium point/s of a nonlinear system to that of its linearized model. **6**

10a.) Find out the sign definiteness of the following quadratic function:

$$x_1^2 + 4x_2^{\ 2} + x_3^2 + 2x_1x_2 - 6x_2x_3 - 2x_1x_3$$ **12**

OR

10b.) Prove that the sufficient and necessary condition for asymptotic stability in the large for a linear system is given by:

$A^T P + PA = -Q$; A, P, Q carry their usual meanings

Where the system is described by the dynamics: $\dot{X} = AX$.

Hence, investigate the stability of the system described by the following equation.

12

$$\dot{X} = AX \ where,$$

$$A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$$