B.E. ELECTRICAL ENGINEERING THIRD YEAR SECOND SEMESTER EXAMINATION, 2018

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Full Marks 100

Time: Three hours (50 marks for each part)

Use a separate Answer-Script for each part

No. of Questions	PART- I	Marks
	Answer any *THREE* questions Two marks reserved for neatness and well organized answers.	
1. (a)	A manufacturer of front lights for automobiles tests a certain number of lamps under a high humidity, high temperature environment, using intensity and useful life as the responses of interest. The following table shows the performance of the tested lamps:	4

		Useful Life	
		Satisfactory	Unsatisfactory
Intensity	Satisfactory	117	3
	Unsatisfactory	8	2

(i) Find the probability that a randomly selected lamp will yield unsatisfactory results under any criteria.

(ii) The customers for these lamps demand 95% satisfactory results. Can the lamp manufacturer meet this demand?

(iii) Given that a randomly selected lamp has satisfactory intensity, what is the probability that it will have an unsatisfactory useful life?

(b)	Suppose small aircrafts arrive at a certain airport according to a Poisson process, with a rate of 8 per hour.	7

(i) What is the probability that exactly 6 small aircraft arrive during a 1 hour period, and at least 6 arrive over the same period?

(ii) What are the expected value and the standard deviation of

[Turn over

Ref No: Ex/EE/T/321/2018

No. of Questions	PART I	Marks
	the number of small aircrafts that arrive during a 90-min period? Derive the expressions used.	
	(iii) What is the time over which 20 small aircrafts are expected to arrive with a probability of 0.8 ?	
(c)	A voltage signal that be represented by a Gaussian random variable (rv) W with a mean of 2V and a standard deviation of 0.2 V, is the input to an amplifier. It gets contaminated by an intrinsically generated random noise U. This noise voltage is also a Gaussian rv with a mean of 0.2 V and a standard deviation of 0.1 V. If the output of the amplifier is $Y = 3W+0.3U$, show that Y is also a Gaussian rv. What are the mean and the variance of Y ?	5
2. (a)	Suppose that a transistor of a certain type, when subjected to an accelerated life test, has a lifetime X (in weeks) that has an exponential distribution with a mean of 24 weeks. Several such identical transistors are put on test simultaneously, all under new condition.	8
	(i) What is the probability that a transistor will last between 10 and 22 weeks? (ii) What is the probability that a transistor will last more than 12 weeks? (iii) Given that a transistor has survived up to 12 weeks, what is the probability that the transistor will survive a further period of 12 weeks. (iv) Suppose the test will actually be terminated after t weeks. What value of t is such that only 0.5% of all transistors would still be operating at termination? Point out the significance of the results of (ii) and (iii).	
(b)	Deduce the expression of the moment generating function for a Gamma distributed rv. There from find the mean and variance of the rv.	4
(c)	Explain with the help of appropriate mathematical derivation, how the Chernoff bound for a random variable can be obtained from a knowledge of its probability density function.	4

Ref No: Ex/EE/T/321/2018

	PART- I	
3. (a)	Define '*Correlation*', '*Covariance*' and '*Correlation Coefficient*' of two random variables. If the random variables are statistically uncorrelated, does it mean that their correlation is zero? Under what condition the correlation and the covariance be same?	4
(b)	Let X and Y be defined by $X = Cos\Theta$ and $Y = Sin\Theta$, where Θ is a random variable uniformly distributed over $(0, 2\pi)$. Are X and Y statistically uncorrelated ? Are they statistically independent ?	6
(c)	Determine the value of c that makes the function $f(x,y) = ce^{-2x-3y}$ a valid joint probability density function over the range $0 \leq x$ and $x < y$. With the proper value of c introduced, determine the following, (i) $P(X<2, Y<2)$ (ii) Marginal cumulative distribution function of X. (iii) Conditional probability density function of Y, given $X = 1$. (iv) $E[Y \mid X=1]$	6
4. (a)	"The magnitude of the cross-correlation function of two jointly wide-sense stationary random processes can never be greater than the product of the root-mean-square values of the random processes". ----- Justify or amend the above statement with relevant derivation.	4
(b)	A zero-mean white noise having a two-sided spectral density of 0.25 V^2 /Hz is applied to the input of a linear time-invariant system whose impulse response is $h(t) = 5\,[\,u(t) - u(t - 0.2)]$ i) Find the mean-square value of the output of the system. ii) Determine the power spectral density of the output. iii) What will the cross-correlation of the output with the input yield? Derive the expressions used for the above computations.	8
(c)	If X_1 and X_2 are two statistically independent random variables and $Y = X_1 + X_2$, verify whether or not the following statements are correct. (i) $F(y) = F(x_1) * F(x_2)$. (ii) $F(y) = f(x_1) * F(x_2)$. The symbols have their usual meaning.	4

[Turn over

	PART- I	
5.	**Write short notes on <u>any two</u> of the following.**	
(a)	**Stationarity and ergodicity of random processes.**	**8+8**
(b)	**Properties of 'Autocorrelation Functions' of wide-sense stationary random processes.**	
(c)	**Bivariate Gaussian distribution function.**	
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 Ref No: Ex/EE/T/321/2018

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No. of Questions	PART- II	Marks
	Answer any *THREE* questions **Two marks reserved for neatness and well organized answers.** **For solving the numerical problems, distribution function tables provided at the may be used with extrapolation/interpolation if necessary**	
1. (a)	In Hypothesis Testing, define the terms, (i) Simple Hypothesis, (ii) Composite Hypothesis, (iii) Type I error, (iv) Type II error, (v) Critical Region (vi) Significance Level.	8
(b)	Discuss one-sided hypothesis test and explain why it is considered to be one-sided.	8
2. (a)	Establish the difference between the maximum likelihood estimator of the standard deviation 'σ' of a normal population and the sample standard deviation 'S' .	8
(b)	Explain with the help of a suitable example, what is 'two-sided percentage confidence interval estimate' of the mean μ of a normal population with known variance σ^2.	8
3. (a)	Discuss Central Limit Theorem and show how the expectation and the variance of the sample mean are related to those of the population.	8
(b)	The amount of water used per day by a person residing in a city has a mean value 0f 145 litres with a standard deviation of 60 litres. If a random sample of 25 persons is taken, determine the approximate probability that the	8

[Turn over

average amount of water consumed per day by the members of the group exceeds 150 litres. Clearly mention the assumption you have to take to solve the problem.

4. (a) With brief introductions to the chi-square and the t-distribution, show how they feature in (i) estimating the confidence interval for the variance of a normal distribution, and in (ii) the hypothesis test concerning the mean of a normal population when the variance is unknown, respectively. 10

(b) The time taken by a CPU to process a certain type of job is normal random variable with mean 20 secs. and standard deviation 3 secs. If a sample of 15 such jobs is observed, what is the probability that the sample variance will exceed 12 sec^2 ? 6

5. (a) Derive the expressions for the Least Square Estimators of the Regression Parameters of a simple linear regression model. Comment on the bias of those estimators and give justification for your comment. 10

(b) A signal having value μ is transmitted from station A, and the value received at station B is normally distributed with mean μ and variance 4. Determine the sample size required to assert a two-sided 95% confidence interval of length 0.1. 6

Distribution Tables:

Normal distribution:

x:	**0.37**	**0.38**	**0.39**	**0.40**	**0.41**	**0.42**	**0.43**	**0.44**
Φ(x):	**.6443**	**.6480**	**.6517**	**.6554**	**.6591**	**.6628**	**.6664**	**.6700**

x:	**1.94**	**1.95**	**1.96**	**1.97**	**1.98**	**1.99**	**2.00**
Φ(x):	**.9738**	**.9744**	**.9750**	**.9756**	**.9761**	**.9767**	**.9778**

Chi-square distribution:

$$\chi^2_{0.05,14} = 23.68\ ;\ \chi^2_{0.1,14} = 21.06\ ;\ \chi^2_{0.25,14} = 17.12$$

$$\chi^2_{0.05,15} = 25.00\ ;\ \chi^2_{0.1,15} = 22.31\ ;\ \chi^2_{0.25,15} = 18.25$$