BACHELOR OF ELECTRICAL ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester)

MATHEMATICS - IF

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use separate answer script for each part.

PART I

Answer any ***five (5)*** questions

1. State and prove Rolle's theorem and give its geometrical interpretation. 10

2. a) Explain whether Rolle's theorem is applicable to the function $f(x) = |x|$ in any interval containing the origin.

   b) If $y = \dfrac{\text{Sin}^{-1}x}{\sqrt{1-x^2}}, |x| < 1$ show that

   $$(1-x^2)y_{n+2} - (2n+3)y_{n+1} - (n+1)^2 y_n = 0$$ 5+5

3. a) State Taylor's theorem (extended to infinity).

   b) Expand in infinite series in x when $f(x) = (1+x)^m$, where m is any number. 2+8

4. Evaluate

   i) $\underset{x\to 0}{\text{Lt}} \left(\dfrac{\tan x}{x}\right)^{1/x}$

   ii) Find a, b such that

   $$\underset{x\to 0}{\text{Lt}} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$$ 5+5

5. a) If $u = x^y$, prove that $\dfrac{\partial^2 y}{dxdy} = \dfrac{\partial^2 y}{\partial y \partial x}$

   b) If $v = 2\cos^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then showed that

   $$x\frac{\partial u}{\partial v} + y\frac{\partial v}{\partial y} + \cot\frac{v}{2} = 0$$

6. If v be a function of $x \infty y$, prove that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}$$ 10

7. a) State the Necessary conditions for Maximum and minimum with two variables.

   b) Find all the maxima & minima of the function

$$4x^2 - xy + 4y + x^3y + xy^3 - 4.$$ 2+8

Part-II

(50 marks)

ANSWER ANY FIVE QUESTIONS

1.(a) A function f is defined on [0,1] by

f(x) = 1, x is rational

=0, x is irrational

Show that f is not integrable on [0,1]. 5

(b) Define Partition, Upper sum, Lower sum, Upper integral and Lower integral. 5

2. (a) Let f: [a,b] → R be a bounded function on [a,b] and P be any partition on [a,b]. Show that L(P,f) ≤ U(P,f). 6

(b) Define norm and refinement of a partition with examples. 4

3. (a) A function f is defined by $f(x) = x^2, x \in [a,b]$ where $a > 0$.

Find $\int_{a^+}^{b} f$ and $\int_{a}^{b^-} f$. Deduce that f is integrable on $[a,b]$. 5

(b) Evaluate the limit $\lim_{n\to\infty}[\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+3n}]$ as an integral. 5

4. (a) Prove that $\frac{\pi^2}{9} < \int_{\frac{\pi}{16}}^{\frac{\pi}{2}} \frac{x}{sinx} dx < \frac{2\pi^2}{9}$. 5

(b) Examine the convergence of $\int_0^1 \frac{x^{p-1}}{1+x} dx$. 5

5. (a) Show that $\int_0^1 x^{m-1}(1-x)^{n-1} dx$ is convergent iff $m > 0, n > 0$. 6

(b) Prove that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. 4

6. (a) Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\,\theta\; cos^{2n-1}\theta d\theta, m > 0, n > 0.$ 5

(b) Prove that $(i)\ \beta\left(\frac{1}{2},\frac{1}{2}\right) = \pi\ \ (ii)\ \gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ 5

7. Let $f: [a,b] \to R, g: [a,b] \to R$ be both integrable on [a,b]. Then f+g is integrable on [a,b] and $\int_a^b f + g = \int_a^b f + \int_a^b g.$ 10