BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

MATHEMATICS - III F

Time : Three hours   Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

(Answer ***any five*** questions)

1. a) Find the Laplace transforms of the following functions :

   i) $\text{Cos}^2 2t$

   ii) $\dfrac{1-e^t}{t}$

   b) If $L\{f(t)\} = \bar{f}(s)$, then prove that

   $$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds,$$ 5

2. a) Find the Laplace Inverse of the following functions

   i) $s\log\left(\dfrac{s-1}{s+1}\right)$

   ii) $\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}$ $2\frac{1}{2}+2\frac{1}{2}$

[Turn over

b) Find the Fourier Transform of $f(x) = \begin{cases} 1-x^2, & |x|<1 \\ 0, & |x|>1 \end{cases}$

and hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\,dx$

$2\frac{1}{2}+2\frac{1}{2}$

3. a) Using Laplace transform, solve the equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}$ Sin t with initial conditions $x(0)=0,\ x'(0)=1$

b) Using Laplace transform, solve the integral equation

$$F(t) = t + 2\int_0^t \cos(t-u)F(u)du$$

5+5

4. a) i) Find the function whose Fourier cosine transform is

$$\hat{f}_c(w) = f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}}\left(a - \frac{w}{2}\right), & \text{if} \quad w < 2a \\ 0, & \text{if} \quad w \ge 2a \end{cases}$$

ii) Find the function whose Fourier transform is

$\hat{f}(w) = e^{-|w|y}$

$2\frac{1}{2}+2\frac{1}{2}$

b) Using Perceval's identities, Prove that

i) $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$

15. a) If f(z) is analytic within and on a closed curve and if a is any point within C, then prove that

$$f^n(a) = \frac{n!}{2\pi i}\int_C \frac{f(z)dt}{(z-a)^{n+1}}$$

b) Evaluate the integral,

$$\int_C \frac{\sin^2 z}{\left(z - \pi/6\right)^3}dt, \text{ where } C:|z|=1.$$

6+4

16. a) Find the nature and location of singularities of the function

$$f(z) = Z\, e^{\frac{1}{z^2}}$$

b) Evaluate $\int_{1-i}^{2+i} (2x+iy+1)dt$ along

$$x = t+1, \quad y = 2t^2 - 1.$$

c) State Residue theorem

4+4+2

b) i) Prove that

$$xP_n'(x) - xP_{n-1}'(x) = nP_n(x)$$

ii) Prove that

$$P_{n+1}'(x) - P_{n-1}'(x) = (2n+1)P_n(x)$$ $2\frac{1}{2}+2\frac{1}{2}$

8. a) Prove that

$$xJ_n'(x) = xJ_{n-1}(x) - J_n(x)$$ 5

b) Prove that

$$2J_n'(x) = J_{n-1}(x) - nJ_{n+1}(x)$$ 5

9. Define regular and irregular singular points of a homogeneous second order linear differential equation. Find the series solution about the point $x = 0$ of the differential equation.

$$9x(1-x)y'' - 12y' + 4y = 0$$ 2+8

10. a) Find series solution about the point $x = 0$ of the differential equation

$$(1+x^2)y'' + xy' - y = 0$$

b) Prove that

$$\int_{-1}^{1} P_m(x)P_n(x)dx = 0, \text{ if } m \neq n$$ 6+4

11. a) Solve the heat conduction equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < l.$$

given that $u(0,t) = 0 = u(l,t)$ and $u(x,0) = f(x)$. 8

b) Write Bessel equation of order n. 2

PART - II

Symbols/Notations have their usual meanings

(Answer ***any five*** questions)

12. a) Show that the function $f(z) = \begin{cases} \frac{(\bar{Z})^2}{Z}, & Z \neq 0 \\ 0, & Z = 0 \end{cases}$

satisfies Cauchy-Riemann equations at (0, 0) but the function is not differentiable at origin.

b) Show that $\lim_{Z \to \infty} \frac{1}{Z^2} = 0$ 7+3

13. a) Find an analytic function W = u + iv = f(z) of which real part u(x, y) is $e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$

b) Suppose f(z) be an analytic function then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$$ 6+4

14. a) Evaluate $\int_C \frac{e^z}{(z^2 - \pi^2)^2} dz$ where

$C : |z| = 4.$

b) Find the Laurent's series expansion of $\frac{z^2 - 1}{z^2 + 5z + 6}$ about z = 0 in the region $2 < |z| < 3.$ 4+6

ii) $\int_0^\infty \frac{t^2}{(t^2+1)^2} dt = \frac{\pi}{4}$ $2\frac{1}{2}+2\frac{1}{2}$

5. a) If f(t) and $f'(t)$ are Laplace transformable and if $L[f(t)] = F(s)$ then prove that

$$\lim_{S \to \infty} sF(s) = \lim_{t \to \infty} f(t).$$ 5

b) State the convolution theorem. Hence find the Inverse Laplace transform $\frac{S}{(S^2 + a^2)^2}$. 1+4

6. a) Find the Fourier transform of

$$f(x) = 1 - x^2, \ |x| < 1.$$ 5

b) Find the Fourier cosine transform of

$$f(x) = \begin{cases} \text{Cos}x, & 0 < x < a \\ 0, & x \geq a. \end{cases}$$ 5

7. a) Establish the relation :

$$(1 - 2xt + t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x),$$

$|x| \leq 1, \ |t < 1$

where $P_n(x)$ is Legendre Polynomial of order n. 5