Ref. No. : EX/EE/MATH/T/112/2018(S)

B.E. ELECTRICAL ENGINEERING SUPPLEMENTARY EXAMINATION 2018

(1ST Year, 1st Semester)

Mathematics – IF

Time: Three Hours

Full Marks: 100

(50 marks for each Part)

Use separate answer script for each Part

PART – I

Answer ***any five*** questions

1\. a) Define norm and refinement of a partition. State Darboux theorem associated with Riemann Integration. 4+6=10

b) Find the upper and lower integral of the following function and prove that it is not Riemann integrable, where $f:[0,1] \to \mathbb{R}$ by $f(x) = 1,\ x\ is\ rational$

$= -1,\ x\ is\ irrational$

2\. a) Test the convergence of the integral $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)}$ 5+5=10

b) Show that the improper integral $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$

3\. a) If n is a positive integer, prove that $2n\Gamma\left(\frac{n+1}{2}\right) = 1.3.5 \dots\dots (2n-1)\sqrt{\pi}$ 3+2+5=10

b) Evaluate $\int_0^{\infty} t^{-\frac{3}{2}}(1-e^{-t})dt$

c) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

4\. Test the convergence of the following series : 6+4=10

a) $\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \cdots \infty,\ x > 0.$

b) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$

5\. a) Show that the series $\frac{2}{1} + \frac{3}{4} + \frac{4}{9} + \cdots + \frac{n+1}{n^2} + \cdots \infty$ is divergent. 5+5=10

b) Test convergence of the series $\sum_{n=1}^{\infty} \frac{2n-1}{\{n(n+1)(n+2)\}}$

6\. a) Show that $\int_1^2 \int_0^{\frac{y}{2}} y\, dy\, dx = \int_1^2 \int_0^{\frac{x}{2}} x\, dx\, dy$. 4+6=10

b) Find the area by double integration the region bounded by circle $x^2 + y^2 = a^2$.

7\. Show that every continuous function is Riemann Integrable. Is the converse true? Give proper justification example. 10

BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester, Supplementary)

MATHEMATICS - I F

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-script for each part.

PART - II

Answer any *five (5)* questions.

1. a) If $p^2 = a^2\text{Cos}^2\theta + b^2\text{Sin}^2\theta$, prove that $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$

 b) If $y = a\text{Cos}(\log x) + b\text{Sin}(\log x)$, show that $\frac{x^2d^2y}{dx} + \frac{xdy}{dx} + y = 0$ 7+3

2. If $y = \frac{\text{Sin}^{-1}}{\sqrt{(1-n^2)}}$ where $1 < x < -1$ and $\pi/2 < \text{Sin}^{-1}x < \pi/2$, prove that

 $(1-x)^2 y_{n+1} - (2n+1)xy_n - n^2y_{n-1} = 0$

 Assuming that y can be expandable in assending power series of x

 $a_1 + a_2x + a_3x^2 + \cdots$

 prove that $(n+1)a_{n+1} = n_{an-1}$, and hence obtain the general term of the expansion.

3. a) Prove that

 $$\text{Sin}ax = ax - \frac{a^3x^3}{3!} + \frac{a^5x^5}{5!} \cdots + \frac{a^{n-1}x^{n-1}}{(n-1)!}\text{Sin}\left(\left(\frac{n-1}{2}\right)\pi\right) + \frac{a^nx^n}{n!}\sin(a\theta x + \frac{n\pi}{2})$$

 b) A function is defined as

 $$f(x) = e^{-1/x^2}, \quad x \neq 0$$
 $$= 0 \qquad x = 0$$

 can be expandable in ascending power of x by maclaranin's theorem. 5+5

[Turn over

4. Discuss the maxima and minima of the function

$$u = \operatorname{Sin}x \cdot \operatorname{Sin}y \cdot \operatorname{Sin}z$$

where x, y, z are the angles of the triangle hence $x + y + z = \pi$ 10

5. a) Evaluate

$$\lim \frac{e^{x} - e^{-x} - 2\log(1+x)}{x\operatorname{Sin}x} \text{ where } x \to 0$$

b) Find the values of a & b in order that

$$\lim_{x\to 0} \frac{x(1 + a\operatorname{Cos}x) - b\operatorname{Sin}x}{x^{3}}$$

5+5

6. a) If $u = e^{xyz}$, show that $\frac{\partial^{3}u}{\partial x \partial y \partial z} = (1 + 3xyz + x^{2}y^{2}z^{2})e^{xyz}$

b) If $u = \log(x^{3} + y^{3} + z^{3} - 3xyz)$ show that

$$\frac{\partial^{2}u}{\partial x^{2}} + \frac{\partial^{2}u}{\partial y^{2}} + \frac{\partial^{2}u}{\partial z^{2}} = \frac{-3}{(x+y+z)^{3}}.$$

7. a) z is a function of x & y, prove that if $x = e^{u} + e^{-v}$, $y = e^{-u} - e^{v}$,

then $\frac{\partial x}{\partial u} - \frac{\partial z}{\partial u} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

b) H is a homogeneous function of x, y, z of degree n, prove that

$$x\frac{\partial H}{\partial x} + y\frac{\partial H}{\partial y} + z\frac{\partial H}{\partial z} = xz$$

5+5