Bachelor of Electrical Engineering, Examination 2018

(1st year, 1st Semester (Supplementary))

MATHEMATICS IIF

Use a separate Answer-Script for each part.

Time: Three Hours **Full Marks: 100**

Part - I

(Answer question no. 6 and any three from rest)

(Symbols/Notations have their usual meanings)

1 a) If $\vec{a} = \text{Sin}\,x\,\hat{\imath} + \text{Cos}\,x\,\hat{\jmath} + x\,\hat{k}\,, \vec{b} = \text{Cos}\,x\,\hat{\imath} - \text{Sin}\,x\,\hat{\jmath} - 3\,k$ and $\vec{c} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, then, find,

$\frac{d}{dx}\{\vec{a} \times (\vec{b} \times \vec{c})\}$ at $x = 0$. 6

b) Find $\vec{\nabla}.\vec{F}$, where $\vec{F} = \vec{\nabla}(x^3 + y^3 + z^3 - 3xyz)$ 6

c) Prove $\iint_S r^5\,\vec{n}\,dS = \iiint_v 5r^3\,\vec{r}\,dv$ 4

2 a) Find $\overline{\nabla} \times (\bar{r}f(r))$ where $f(r)$ is differentiable. 6

b) Show that the vector $\vec{v} = 2xyz^3\,\hat{\imath} + x^2z^3\,\hat{\jmath} + 3x^2yz^2\,\hat{k}$ is irrotational. Find the scalar potential φ such that $\vec{v} = \vec{\nabla}\varphi$ 6

c) Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2$ at $(2, -1, 2)$ in the direction $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. 4

3 a) Show that under rotation of rectangular axes the origin remaining the same, the vector differential operator $\vec{\nabla}$ remains invariant. 6

b) Show that $\vec{\nabla}.(\vec{F} \times \vec{G}) = \vec{G}.\vec{\nabla} \times \vec{F} - \vec{F}.\vec{\nabla} \times \vec{G}$

Hence show that, if the vectors $\vec{F}$ and $\vec{G}$ are irrotational, show that $\vec{F} \times \vec{G}$ is solenoidal. 6

c) Evaluate $\int_2^3 (\vec{r} \times \frac{d^2\vec{r}}{dt^2})\,dt$, where $\vec{r}(t) = t^3\vec{\imath} + t^2\vec{\jmath} + t\vec{k}$. 4

4 a). Evaluate $\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\,ds$, where $\vec{F} = y\vec{\imath} + (x - 2xz)\vec{\jmath} - xy\vec{k}$ and S is the surface of the of the sphere $x^2 + y^2 + z^2 = z^2$ above xy –plane. 8

b). Verify Stokes' theorem for $\bar{F} = (x^2 + y^2)\,\hat{\imath} - 2xy\,\hat{\jmath}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. 8

[Turn over

5 a). Derive an expression for $\overline{\nabla}\emptyset$ in spherical polar coordinates. Hence prove that

$\overline{\nabla}(\cos\theta) \times \overline{\nabla}\varphi = \overline{\nabla}\left(\frac{1}{r}\right),\ r \neq 0.$ 8

b). If u, v, w are orthogonal curvilinear co-ordinates show that $\frac{\partial \vec{r}}{\partial u}$, $\frac{\partial \vec{r}}{\partial v}$, $\frac{\partial \vec{r}}{\partial w}$ and $\vec{\nabla} u, \vec{\nabla} v, \vec{\nabla} w$ are reciprocal system of vectors. 8

6. Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C x\,dy - y\,dx.$ 2

Ref. No. Ex/EE/Math/T/113/2018

BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING EXAMINATION – 2018

1st year, 1st semester
Mathematics-II F

Time: 3 hours **Full Marks: 100**

Use a separate answer-script for each Group.

Group – B (50 Marks)

(Symbols have their usual meaning)

Answer ***any five*** questions from Group – B.

7. (a) Express $\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \beta\gamma & \gamma\alpha & \alpha\beta \end{vmatrix}$ as a product of three linear factors.

(b) Prove that

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

8. (a) Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 3 & 5 & 8 \\ 4 & 8 & 0 \end{pmatrix}$.

(b) Investigate for what values of λ and μ the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have (i) no solution and (ii) a unique solution

9. (a) Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -1 & -4 & -1 \end{pmatrix}$$

(b) Use Cayley-Hamilton theorem to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

10. (a) Find a real Orthogonal matrix of order 3 having the elements $\frac{1}{3}$, $-\frac{2}{3}$, $\frac{2}{3}$, as the elements of a column.

(b) Solve, by matrix method, the equations

$$x + y + z = 8$$
$$x - y + 2z = 6$$
$$3x + 5y - 7z = 14.$$

11. (a) Solve the differential equation

$$\cos x \frac{dy}{dx} - y \sin x = y^3 \cos^2 x$$

(b) Find the integrating factor of the differential equation

$$(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$$

12. (a) Find the general solution of the differential equation

$$(D^4 + 2D^2 + 1)y = x^2 \cos x \; ; \; D = \frac{d}{dx}$$

(b) Find the particular integral of

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10(x + x^{-1})$$

13. (a) Find the particular integral of

$$\frac{d^2y}{dx^2} + a^2y = \tan ax$$

(b) Solve

$$(x^3 + xy^4)dx + 2y^3dy = 0$$

14. (a) Use method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + y = x \sin x$$

(b) For what value of k, $(c_1 + c_2 lnx)/x$ is the general solution of the differential equation $x^2 \frac{d^2y}{dx^2} + kx \frac{dy}{dx} + y = 0$?