(b) The overall percentage of failure in a certain examination is 40. Find the probability that out of a group of 6 candidates at least 4 passed the examination. 5+5

13. (a) Define a mutually exclusive and exhaustive events.

(b) Suppose a box contains 3 white balls and 2 black balls. Let E_1 be the event "first ball is black" and E_2 the event "second ball is black", where the balls are not replaced after being drawn. Find $P(E_1)$, $P(E_2/E_1)$ and $P(E_1E_2)$.

Hence show that E_1 and E_2 are dependent.

(c) From a population of large number of men with standard deviation 5, a sample is drawn and the standard error is found to be 0.5. What is the sample size ? 2+5+3

14. (a) For a population of six units, the values of a characteristic x are 3, 9, 6, 5, 7, 10.

Consider all possible samples of size two from the above population, show that the mean of the sample means is exactly equal to the population mean.

(b) Prove that the standard deviation for a Binomial distribution $B(n,p)$ is $\sqrt{npq}$. 6+4

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Ex./CON/MATH/T/125/2018

BACHELOR OF CONSTRUCTION ENGG. EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - II E

Time : Three hours Full Marks : 100

Symbols/Notations have their usual meanings.

Answer any ***ten*** questions.

1. (a) Show that the necessary and sufficient condition for two vectors to be perpendicular is that their scalar product should be zero.

(b) Show that the three points $\hat{i}-2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$ and $-7\hat{j}+10\hat{k}$ are collinear.

(c) Show that the line joining the mid-points of the two sides of a triangle is parallel to the third side and half of its length. 3+3+4

2. (a) If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, show that $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$. Also find the angle between $2\vec{a}+\vec{b}$ and $\vec{a}+2\vec{b}$.

(b) Given $\vec{a}=2\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=6\hat{i}-3\hat{j}+2\hat{k}$, find $\vec{a}\times\vec{b}$ and the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. Also find the sine of the angle between $\vec{a}$ and $\vec{b}$. 5+5

(Turn Over)

3. (a) Find the equation of the line through the point (1,2,–1) and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$.

(b) Find the shortest distance between the two lines

$$\frac{x-x_1}{\ell_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{\ell_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$$

5+5

4. (a) Form a partial differential equation by eliminating the arbitrary constants a, b and c from $z = a(x+y) + b(x-y) + abt + c$.

(b) Form a partial differential equation by eliminating the arbitrary function f from $f(x+y+z, x^2+y^2-z^2)=0$.

4+6

5. (a) Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = 2\sin y$ when x= 0, and z = 0 when y is an odd multiple of $\frac{\pi}{2}$.

(b) Solve :

(i) $y^2 p - xy\, q = x(z-2y)$

(ii) $xp - yq = y^2 - x^2$

4+(3+3)

(b) Find the cumulative frequencies (less than and greater than type) of the following distribution :

Height in inches:	50-54,	55-59	60-64	65-69	70-74
Frequency :	12	16	10	8	4

4+6

11. (a) A random variable has the following probability distribution :

x :	3	4	6	7
p :	0.1	0.4	0.3	0.2

Find the expectation and standard deviation of the random variable.

(b) Prove that $Var(ax+b) = a^2\, Var(x)$.

(c) Arithmetic mean and standard deviation of a Binomial distribution are 4 and $\sqrt{\frac{8}{3}}$ respectively. Find the values of its parameters.

4+3+3

12. (a) Show that the Poisson distribution is an approximation of Binomial distribution when ‘p’ is small and ‘n’ is large but np is finite.

6. Obtain the various possible solutions of the one-dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},$$

by the method of separation of variables. Identify the solution which is appropriate wih the physical nature of the equation. Justify your answer. 10

7. Solve the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial t^2}, \quad 0 \le x \le L, \ t > 0$$

subject to the conditions :

u(0,t) = 0 and u(L,t) = 0 for all t,

$u(x,0) = f(x)$ and $\left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$ for $0 \le x \le L$. 10

8. (a) Solve the following by graphical method :

Minimize $z = 3x+5y$

subject to $2x + 3y \ge 12$

$-x + y \le 3$

$x \le 4$

$y \ge 3$.

(Turn Over)

(b) Find the feasible region and optimal solution(s) of the problem

Maximize $z = 2x_1 - x_2$

subject to $x_1 - x_2 \le 1$

$x_1 \le 3$, and $x_1, x_2 \ge 0$. 5+5

9. (a) For what value of 'a' will the function $f(x) = ax$; $x = 1,2,3,\ldots, n$ be the probability man function of a discrete random variable x ? Find also the mean and variance of x.

(b) The p.d.f. of a continuous random variable is

$y = K(x-1)(2-x), \ 1 \le x \le 2.$

Determine

(i) The value of the constant K.

(ii) Cumulative distribution function,

(iii) The probability that x lies between $\frac{5}{4}$ and $\frac{3}{2}$.

6+(1+1+2)

10. (a) Prepare a frequency distribution table with class intervals 60-69, 70-79, 80-89 and so on from the following data relating weight of 20 apples :

93, 87, 79, 94, 67, 78, 95, 73, 69, 68, 130, 95, 82, 103, 117, 89, 97, 83, 108, 96