6. (a) State and prove Euler's theorem for homogeneous functions of degree n. 2+7

(b) If u = e^{xyz}, then show that

$$\frac{\partial^3 u}{\partial x\, \partial y\, \partial z} = \left(1 + 3xyz + x^2 y^2 z^2\right) e^{xyz} \qquad 5$$

(c) If $u = \cos^{-1}\left\{(x+y)/\left(\sqrt{x} + \sqrt{y}\right)\right\}$ then show that

$$xU_x + yU_y + \frac{1}{2}\cot u = 0. \qquad 6$$

7. (a) When a function is called solenoidal and irrotational ? 3

(b) Let f (x, y, z) = $x^2 + y^2 + xz$. Find the directional derivatives of f at the point P(2, –1, 3) in the direction of the vector $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$. 7

(c) Let $\vec{F}(x, y, z) = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$. Evaluate $\iiint_V \vec{F}.dv$ where V is the region bounded by the surfaces x = 0, y = 0, z = x^2, y = 6, z = 4. 10

8. (a) State Green's theorem. Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int_C (xdy - ydx)$. 10

(b) State Gauss Divergence theorem. Use it to evaluate $\iint_S \vec{F}.d\vec{s}$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. 10

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Ex./CON/MATH/T/112/2018(S)

BACHELOR OF CONSTRUCTION ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester, Supplementary)

Mathematics - I E

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

1. (a) Show without expending

(i)
$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

(ii)
$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

(iii)
$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

5+5+5

(b) Solve by Crammer's rule x + 2y + 3z = 6, 2x + 4y + z = 7, 3x + 2y + 9z = 14. 5

2. (a) Find the inverse of $\begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{pmatrix}$. 5

(b) Find the rank and normal form of the matrix

$$\begin{pmatrix} 1 & -4 & 5 & 1 \\ 1 & 2 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ 1 & 8 & -7 & 1 \end{pmatrix}.$$ 7

(c) Solve (if possible)

(i) $x + y + z + \omega = 0$

$x + 3y + 2z + 4\omega = 0$

$2x + z - \omega = 0$

(ii) $x - 2y + z - \omega = -1$

$3x - 2z + 3\omega = -1$

$5x - 4y + \omega = -4$ 5+3

3. (a) Find the eigen values and eigen vectors of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$ 9

(b) Without integrating prove that

(i) $\int_0^{\pi} x \log \sin x \, dx = \frac{\pi^2}{2} \log \frac{1}{2}$

(ii) $\int_0^{\pi} x \sin \sin x \cos^2 x \, dx = \frac{\pi}{3}$ 7+4

4. (a) State and prove Darbaux's theorem. 1+6

(b) Evaluate $\int_a^b x^m dx$ from definition of Riemann integral as limit of sums. 5

(c) Test the convergence

(i) $\int_0^{\pi/2} x^m \operatorname{cosec}^n x \, dx$ (ii) $\int_0^{\infty} \frac{dx}{(1+x)\sqrt{x}}$ 3+5

5. (a) State Rolle's theorem. Explain with reasons whether Rolle's theorem is applicable to $f(x) = \tan x$ in $[0, \pi]$. 5

(b) State and prove Lagrange's Mean Value Theorem. Give its geometric interpretation. 1+5+2

(c) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ exists then find the value of 'a' and also find the limit. 7