Ex./CON/MATH/T/112/2018

BACHELOR OF CONSTRUCTION ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester)

Mathematics - I E

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

1. (a) Show withous expanding

(i) $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

(ii) $$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

(b) Solve by Crammer's rule $2x+3y+4z=20$, $x-5y+6z=9$, $3x+4y-5z=-4$.

(c) Find the inverse of

$$\begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{pmatrix}.$$

(5+5)+5+5

(Turn over)

2. (a) Find the rank and normal form of the matrix

$$\begin{pmatrix} 2 & 0 & 2 & 2 \\ 3 & -4 & -1 & -9 \\ 1 & 2 & 3 & 7 \\ -3 & 1 & -2 & 0 \end{pmatrix}$$ 6

(b) Solve the system of equations

(i) $x + y + z + u = 0$ (ii) $x - 2y + z - w = -1$

$3x + 4y - z = 0$ $3x - 2z + 3w = -4$

$x + 2y - 3z + u = 0$ $5x - 4y + w = -3$ 5+3

(c) Find the eigen values and eigen vectors of

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$ 6

3. (a) Without integrating prove that

(i) $\int_0^{\pi} x \sin x \cos^2 x \, dx = \frac{\pi}{3}$

(ii) $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$ 5+5

(b) State and prove Fundamental Theorem of integral calculus. 1+7

8. (a) State Green's theorem in a plane. 2

(b) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int_C (x\,dy - y\,dx)$. 8

(c) State the Gauss, Divergence Theorem. Use it to evaluate $\iint_S \vec{F} \cdot \overrightarrow{ds}$ where $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 2+8

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(c) Give an example of a function which is not Riemann integrable. 2

4. (a) State and prove First Mean Value Theorem of integral calculus. 1+4

(b) If f is Riemann integrable on $[a, b]$ then prove that $|f|$ is also so on [a,b] and $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|\,dx$. 6

(c) Evaluate (if possible)

(i) $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ (ii) $\int_{-\infty}^{\infty} \frac{xdx}{x^4+1}$ 5+4

5. (a) Let S and T be any two non-empty sets. Show that there is a bijection between S x T and T x S. 10

(b) State Rolle's Theorem. If f(x) = tan x in a domain containing 0 and π, explain whether Rolle's theorem is applicable to f in $[0, \pi]$? 5

(c) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2} f''(\theta h)$, $0 < \theta < 1$, find the value of θ when $h = 1$ and $f(x) = (1-x)^{5/2}$. 5

6. (a) State and prove Euler's theorem for homogeneous functions of degree n. 2+7

(b) If $f(x,y) = xy \cdot \frac{x^2-y^2}{x^2+y^2}$, $(x,y) \neq (0,0)$

$= 0$, $(x,y) = (0,0)$

then show that $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$ at $(0,0)$. 6

(c) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ exists then find the value of 'a' and also find the limit. 5

7. (a) Define directional derivative of a scaler function in the direction of a fixed vector. 2

(b) Let $\phi(x,y,z) = x^2 + y^2 + xz$. Find the directional derivative of ϕ at the point P(2,–1,3) in the direction of the vector $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$. 8

(c) Let $\vec{F}(x,y,z) = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$, evaluate $\iiint_V \vec{F} \cdot dv$ where V is the region bounded by the surfaces $x = 0$, $y = 0$, $y = 6$, $z = x^2$ and $z = 4$. 10