B.E. Computer Science & Engineering Examination 2018

Second year First Semester

NUMERICAL METHODS

Time : 3 hours.

Full Marks : 100

Answer question no.1 and any 4 from the rest.
All parts of same question should be answered together.

1. a) Define round-off and truncation errors. 2
   b) Draw a comparison between regula falsi method and secant method. 3
   c) Show that Newton- Raphson formulato find $\sqrt{a}$, a>0, can be expressed in the form

   $$x_{n+1} = [\, x_n + a/ x_n \,] / 2$$ 4

   d) Write down the expressions for truncation errors for Trapezoidal method, Simpson's ⅓ rule and Simpson's ⅜ rule. 3
   e) Define Δ, and E. Hence prove that

   $$\Delta - \nabla = -\Delta\nabla$$ 3

   f) Why Gauss-Seidel method is better than Jacobi's method for solution of linear simultaneous equations? 1
   g) What is limitation of Jacobi's method for finding the eigenvalues of a symmetric matrix? 2
   h) Modify Newton- Raphson iteration formula for solution of a nonlinear equation with multiple roots at a point. 2

2. a) Describe secant method for solution of non-linear equations. 4
   b) Derive the order of convergence for the above method. 5+5
   c) Solve the following equation using Newton- Raphson method. 6

   $$x \log_{10} x - 1.2 = 0$$

   Solution is required to be corrected upto 4 decimal places. Choose your own initial approximation.

3. a) Discuss Jacobi's iterative method for finding the roots of linear simultaneous equations. 6
   b) Write down the method in matrix notation. 4
   c) Hence find the convergence of the method. 4
   d) Solve the following system of equations by LU decomposition method. 6

   $$x + y + z = 9$$
   $$2x - 3y + 4z = 13$$
   $$3x + 4y + 5z = 40$$

[Turn over]

4. a) Define the terms eigenvalue and eigenvector. 2

b) Find all the eigenpairs (λ_i , X_i) of the following matrix by Jacobi's method.

$$A = \begin{matrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{matrix}$$

8

c) Given the following table of values:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

Scale the x-values so that their mean becomes zero.
Obtain a least squares fit of the following form to the scaled tabular values.

$$y = aX^2 + bX + c$$

Finally, restore the unscaled form of the fitted polynomial. 10

5. a) Discuss Adams-Bashforth method for solution of ordinary differential equations for initial value problem. 10

c) Solve the following initial value problem using Euler's method.

$$\frac{dy}{dx} = 3(x + y) \text{ with } y(0) = 1$$

10

Solution is required over [0,1] with h = 0.1.
Calculate the percentage error with the exact solution

$$y = (4e^{3x} - 3x - 1) / 3$$

6. a) Derive Gauss central difference interpolation formulae. 4+4

b) The velocity '*v*' of a particle at a distance '*s*' from a point on its path is given in the following table :

'*s*' in metres	0	10	20	30	40	50	60
'*v*' in m/sec	47	58	64	65	61	52	38

Estimate the time taken to travel 60 metres using Simpson's ⅓ rule. 6

c) Find the inverse of the following matrix using Gauss-Jordan method. 6

$$A = \begin{matrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{matrix}$$

[Turn over]

7\. a) Discuss Romberg's method for evaluating the integral of the following form.

$$\int_b^a f(x)dx$$ 10

b) Evaluate the following integral using Gauss Quadrature formula. Take n = 2, 3 and calculate the errors with respect to the true value.

$$\int_0^1 (1/(1+x^2))\,dx$$ 10

8\. a) Discuss basic principles of Spline interpolation method. 10

b) Describe Bairstow's method for finding complex roots of a polynomial equation. 10
