B.E. Computer Science and Engineering Second year First Semester Supplementary Exam. 2018

NUMERICAL METHODS

Full Marks: 100 Time: 3 hours.

Answer any 5 questions.

1. a) Prove by the method induction

$$\Delta^{m} y_{r} = \nabla^{m} y_{r+m}$$
 5

b) If y = U(x).V(x) then $y[x_0, x_1] = U(x_0).V[x_0, x_1] + V(x_1).U[x_0, x_1]$ 3

c) Use Newton's formula to find the area of a circle of diameter 98 cm from the following area (A) vs. diameter (D) table:

D(cm)	80	85	90	95	100
A (cm ²)	5026	5674	6362	7088	7854

Derive the necessary formula.

12

2. a) Discuss Gauss-Jordan elimination method for matrix inversion.

8

b) Find the inverse of the following matrix.

12

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 4 \end{bmatrix}$$

Also find AA-1.

3. a) Define the terms eigenvalue and eigenvector.

4

b) Prove that if a square matrix is transformed into a pure diagonal form, then the diagonal elements represent the eigenvalues of the matrix.

6

c) Find all the eigenvalues and eigenvectors of the following matrix by Jacobi's method.

10

$$\begin{bmatrix} 1 & \sqrt{3} & 4 \\ \sqrt{3} & 5 & \sqrt{3} \\ 4 & \sqrt{3} & 1 \end{bmatrix}$$

(Turn Over)

4. a) Discuss Newton-Raphson method for solution of non-linear equations. 8 6 b) Derive the order of convergence for Newton-Raphson method c) Show that either of the following iteration formula can be used to find √N by Newton-Raphson method. 6 $r_{i+1} = r_i * (1 + N/r_i^2)/2$ a) Discuss Divided Difference formula for interpolation. 10 b) Given the following tabular values: 6 2 7 17 53 157 857 10 Find x at y=257 using above method. 10 6. a) Discuss curve fitting by the method of least squares. b) Discuss Gauss-Seidel iterative method for finding the roots of a set of linear simultaneous equations. Represent the method using matrix notations. 10 Derive Trapezoidal rule for evaluating the integral $\int_{a}^{b} f(x) dx$ 8 b) Calculate the amount of truncation error involved in this method. 6 c) Evaluate the following integral by the above method. $\int_{1}^{\pi} (1/x) dx$. Take h = 0.2. 6 Calculate the percent error from the actual value of the integral. 8. a) Discuss Modified Euler's formula for solution of differential equations. 6 4 b) Derive the expression for truncation error of the above method. c) Solve the following differential equation by **Euler's** method. $\frac{\partial (y)}{\partial x} = x^2 + y \qquad \text{with } y(0) = 1.0$ Compute the first 5 steps of the solution with step size h = 0.1Compare the results with those obtained from the exact solution 10 $y = 3 e^{x} - x^{2} - 2x - 2$