Ref. No.: Ex/CSE/MATH/T/211A/2018(S)

B.CSE, 2ND YR. 1ST SEMS SUPPLE. EXAM, 2018

Mathematics - IV

Full Marks:100 Time: Three Hours

Answer question number 1 and any six questions from the rest.

1. Find the radius of convergence of the following series. (4)

(a)

$$\sum_{n=1}^{n=\infty} \frac{(n!)^2 x^n}{(2n)!},$$

(b)

$$y = 1 - \frac{n^2}{2!}x^2 + \frac{n^2(n^2-2^2)}{4!}x^4 - \frac{n^2(n^2-2^2)(n^2-4^2)}{6!}x^6 + \cdots,$$

where n is not an integer.

2. (a) Solve the following initial value problem using power series method (10)

$$y'' - xy = 0,$$

where $y(1) = 2$ and $y'(1) = 0$. Write first four non-zero terms of the series.

(b) Find power series solution about origin of the differential equation (6)

$$y' - 4y = 0$$

Express the solution in terms of elementary function.

3. Determine all singular points of the given differential equation. Classify those singularity. Also find Frobenius series solution of the following differential equation about $x = 0$ (16)

$$2x(1-x)y'' + (1-x)y' + 3y = 0.$$

Write first three non-zero terms in each series.

4. (a) Find general solution of the differential equation (8)

$$y'' + 7y' + 12y = e^x \sin 2x.$$

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(b) Use the method of variation of parameters to find a particular integral of the differential equation (8)

$$y'' + 4y = \cos x.$$

5. (a) Prove that (10)

$$P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n,$$

where $P_n(x)$ is the Legendra polynomial of degree n. Hence find the expressions for P_0, P_1, P_2 and P_3.

(b) Express $x^3 + x + 1$ in terms of Legendre polynomials. (6)

6. (a) State and prove orthogonality property of Chebyshev polynomials. (6)

(b) Find all zeros and extremum points of Tchebychef polynomial of degree n, denoted by $T_n(x)$. Find also the extremum values of it. (6)

(c) Prove that (4)

$$\frac{1-xt}{1-2xt+t^2} = \sum_{n=0}^{\infty} T_n(x)x^n$$

7. (a) Show that $f(z) = |z|^2$ is differentiable only at $z = 0$ using concept of limits. (4)

(b) Use Cauchy Riemann equations to determine the analytic function $f(z) = u + iv$, where $u + v = e^x(\cos y + \sin y)$ (6)

(c) Show that the function $u(x, y) = 2x + y^3 - 3x^2y$ is harmonic. Find its conjugate harmonic function $v(x, y)$. (6)

8. (a) Evaluate (4)

$$\int_0^{2\pi} \frac{dt}{(t+i)^2}.$$

(b) Find the value of the integral $\int_C |z|^2 dz$, where C is the boundary of the square with vertices at (0,0), (2,0), (2,2) and (0,2) in that order. (8)

(c) Compute the residues at the singular points of the function (4)

$$f(z) = \frac{z}{(z+1)(z-2)}.$$

9. (a) Use Cauchy integral formula to show that (4)

$$\int_C \frac{dz}{2-\bar{z}} = \frac{\pi i}{2}, \quad \text{where} \quad C : |z| = 1.$$

(b) Find all possible Taylor's and Laurent series expansions of the function $f(z) = 1/(1-z)$ around $z = 0$. (6)

(c) Show that the function $f(z) = \frac{1}{z^2-1}$ has simple poles at $z = 1$ and $z = -1$. (6)

10. (a) Find Fourier series expansion of the following function of period 2π (8)

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ 0, & 0 \le x < \pi \end{cases}$$

(b) Find Fourier cosine and sine series of the function $f(x) = 1, 0 \le x \le 2$. (8)