B.E.COMPUTER SCIENCE AND ENGINEERING SECOND YEAR FIRST SEMESTER (OLD) SUPPLEMENTARY EXAM- 2018

MATHEMATICS- VD

Full Marks: 100 **Time: Three Hours**

(Answer any five questions)

(Symbols/Notations have their usual meanings)

1) Solve the following differential equations

a) $(2D^3 - 3D^2 + 1)y(x) = 1 + e^x$ where $D = \frac{d}{dx}$ 5

b) $(D^3 - 3D^2 + 3D - 1))y(x) = e^x(x+1)$ 5

c) $(D^2 - D + 2)y(x) = 5\cos x$, given that $y = Dy = o\ when\ x = 0$ 5

d) $(D^2 + 5D - 6)y(x) = \sin 4x \sin x$ 5

2a) Solve the following differential equation by the method of variation of parameters

$$(D^2 + 4)y(x) = \sin 2x$$ 8

b) Solve the following differential equation

$$(x^2D^2 + 4xD + 2)y(x) = logx$$ 8

c) Using Rodrigue's formula , find the polynomial expression for in terms of $P_4(x)$. 4

3a) Solve the following differential equation about $x = 0$

$$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1)y = 0$$ 10

b) Show that $P_n(x) = \frac{1}{n!\ 2n}\frac{d^n}{dx^n}(x^2-1)^2$ 10

4a) Prove that

i) $P_{n+1}^{/}(x) - x\,P_n^{/}(x) = (n+1)P_n(x)$

ii) Show that $\int_{-1}^{1} f(x)\,P_n(x)dx = \frac{(-1)^n}{2^n\,n!}\int_{-1}^{1}(x^2-1)^n f^n(x)dx$ 12

b) Prove that

i) $\frac{\sqrt{1-x^2}}{1-2xt+t^2} = \sum_0^\infty U_{n+1}(x)t^n$

[Turn over

ii) $(1-x^2)\,T_n^{/}(x) = \frac{n}{2}\,[T_{n-1}(x) - T_{n+1}(x)]$ 8

5 a) State Dirichlet's condition for a Fourier series expansion of function. Find the Fourier series for the function $f(x)$ defined by

$$f(x) = x - x^2 \ , -\pi \le x \le \pi$$

Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi}{12}$ 3+7

b) Obtain the half range Cosine series for

$$f(x) = x \ for \ 0 \le x \le \frac{l}{2}$$

$$= l - x \ \ for \ \frac{l}{2} \le x \le l$$ 10

6a) A periodic function of period 4 is defined as

$$f(x) = IxI \ ; \quad -2 \le x \le 2.$$

Find its Fourier expansion. 10

b) A periodic function of period 2π is defined as

$$f(x) = x + \pi \quad for \ 0 \le x \le \pi$$

$$= -x - \pi \ for \ \ -\pi \le x \le 0$$

Find its Fourier series. 10

7a) Show that the function

$$f(z) = \sqrt{|xy|}$$

Is not regular at the origin although the Cauchy-Riemann equations are satisfied at that point. 8

b) If $f(z) = u(x,y) + v(x,y)$ is an analytic function of $z = x + iy$, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) If^{/}(z)I^2 = 4\, If^{/}(z)I^2$$ 8

c) If $u(x,y) = x^2 + y^2$ is the real part of an analytic function $f(z) = u(x,y) + i\,v(x,y)$ find $v(x,y)$. 4

8a) Expand the $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurant series valid for $i)\ IzI > 3, ii)\ 0 < Iz + 1I < 2,$ 4

b) Determine the poles and the residue at each point of the function

$$f(z) = \frac{z^2}{(z-1)^2\,(z+2)}$$ 8

c) Evaluate $\int_0^{\infty} \frac{\sin x}{x}\, dx$ 8