7. Write down the joint probability density function of a bivariate normal random vector (X, Y) with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ where $\mu_1, \mu_2 \varepsilon \mathbb{R}$, $\sigma_1^2, \sigma_2^2 > 0$, $|\rho| \leq 1$,

   a) Find Var(X), Var(Y), Cov(X,Y) and the correlation coefficient between X and Y.

   b) Find the regression equation of Y on X.

Bachelor of Engineering in Computer Science & Engineering Examination, 2018

(2nd Year, 2ndSemester)

Mathematics - V

Time : Three hours Full Marks : 100

Attempt ***any five*** questions.

Each question carries twenty (20) marks.

1. Define a random experiment and the sample space associate with it. Give an example to clarify your definition. Define probability classically. Mention two major drawbacks of classical probability. Define probability axiomatically to explain with an example.

   An unbiased coin is repeated tossed (independent) until a Head appears. At this point, the experiment is stopped and the no of tosses noted.

   Let X denote the (random) number of tosses required to get the first Head.

   Write down the sample space and find E(X), V(X). What is the name of X.

2. a) A committee of 5 is to be selected at random out of 7 women and 13 men. Find the probability that two selected committee consists of 3 men and 2 women.

b) n people are in a room. If a year is treated as 365 days, what is the probability that no two of them celebrate their birthday on the same day of the year.

3. a) State and prove Bayes' forumla for conditional probability.

   b) A certain letter is equally likely to be in any one of the three envelopes. To check, a man can quickly perform a test which tells him whether or not the letter is, in fact, in envelop i, is prone to errors and But this quick test has probability α_i (i = 1, 2, 3) of giving a correct result i.e. α_i is the condition probability that the test infers that the letter is in enfolder i given that the letter is actually in envelop i.

   What is the conditional probability that the letter is actually in envelop i given that the quick test said the letter is not in envelop 2.

4. Define a random variable and its characteristic function.

   Prove that if two random variables X and Y are independent then the characteristic function of X+Y is the product of their individual characteristic function.

   Find the mean and variance of a Hypergeometric Random variable.

5. a) State and prove Markov's Inequality.

   b) State and prove Chebyshev's Inequality.

   c) State and prove the Weaklaw of Large Numbers.

   d) If it is known that E(X) = 75, give an upper bound of $P(X \geq 85)$

   If further, it is known that Var (X) = 25

   find a lower bound of $P(65 \leq X \leq 85)$

6. a) Define the cumulative distribution function of a Bivariate Random vector (X, Y) at the point (s, t).

   If F is the cumulative distribution function of (X, Y) and if $c \geq a, d \geq b$, prove that

$$P(a < x \leq c, b < y \leq d)$$

$$= F(c, d) + F(a, b)$$

$$-F(a, d) - F(c, b)$$

   b) Let the joint probability density function of a random vector (X, Y) be given as

$$f(x, y) = \begin{cases} k, & \text{if} \quad 0 \leq x^2 + y^2 \leq 4 \\ 0 & \text{Otherwise,} \end{cases}$$

   Find k, Var (X) and Cov (X, Y).