 Ref. No.: **Ex/CSE/T/224A/2018**

B.E. Computer Science and Engineering
2^{nd} Year, 2^{nd} Semester Examination, 2018
Graph Theory and Combinatorics

Full Marks: 100 Time : 3 Hr

Answer Any Five Questions

Write answers to the point. Make and state all the assumptions (wherever required).

PARTS OF THE QUESTION SHOULD BE ANSWERED TOGETHER

Q 1) (a) Define Hamiltonian and Eulerian graphs. Prove the complete graph $K_{3,3}$ is Hamiltonian but not Eulerian. $(2+2+4=8)$

(b) Explain the following (i) Planar Graphs (ii) Complete Bipartite Graphs $(3+3=6)$

(c) If a graph has girth g and diameter d, show that $d \geq \lfloor \frac{g}{2} \rfloor$ (6)

Q 2) (a) Show that there are only five regular polyhedra (**Polyhedra: a solid figure with many plane faces, typically more than six.**) [*Note : Use Euler's Formula for Planar Graphs*] (10)

(b) Show that the Peterson Graph is non-planar by establishing that it has a $K-subgraph$ (5)

(c) Prove that every graph of order 6 with chromatic number 3 has at most 12 edges. (5)

Q 3) (a) The degree of the vertices of a certain tree T of order 13 are 1,2 and 5. If T has exactly three vertices of degree 2, how many end-vertices/terminals does it have ? (6)

(b) Apply both Kruskal's and Prim's Algorithms to find a minimum spanning tree in the weighted graph of Figure 1. In each case, show how this tree is constructed. $(7+7=14)$

Q 4) (a) For each of the following in Figure 2, decide whether or not the graph is planar. If it is, draw a planar representation of the graph. If not, show that it is not planar. $(4 \times 2 = 8)$

(b) Prove that the Peterson graph G is non-Hamiltonian. (4)

(c) The graph *n-cube*, denoted by Q_n is defined as follows: (8)
$V(Q_n) = \{(a_1, a_2, \ldots, a_n) | a_i = 0\backslash 1\}$. Two vertices $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n) \in V(Q_n)$ are adjacent in Q_n i.f.f. for exactly one i $(1 \leqslant i \leqslant n)$, $a_i \neq b_i$. Draw Q_1, Q_2, Q_3 and show that Q_n is a regular graph

Q 5) (a) Verify that the two graphs K_5 and $K_{3,3}$ have the following properties. $(3 \times 5 = 15)$

(i) Both are regular.

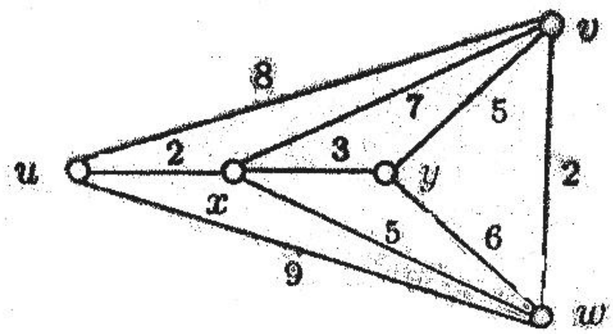

Figure 1: Figure for Problem (3b)

Figure 2: Figures for Problem (4a)

(ii) Both are non-planar.

(iii) The graph obtained be removing exactly one edge or a vertex from K_5 or $K_{3,3}$ is a planar graph.

(iv) K_5 is a non-planar graph with the smallest number of vertices.

(v) $K_{3,3}$ is the non-planar graph with the smallest number of edges.

(b) Prove that the Kruskal's Algorithm always constructs an optimal spanning tree for a weighted undirected graph. (5)

Q 6) (a) Let $G(V, E, \gamma)$ be a connected graph, where V is a set of vertices, E is set of edges and γ is a function of the vertices which $e \in E$ joins. In the graph, G every vertex has even degree. Show that G has no cut edges. (5)

(b) Prove that every tree has at most one perfect matching. (5)

(c) Show that the Ramsey number $R(K_4, K_3) = 9$. (10)

Q 7) (a) Show that the maximum number of edges in a bipartite graph on $|V|$ vertices is $[\frac{|V|^2}{4}]$. (10)

Note : $[x]$: *The greatest Integer not greater than the real number* x

(b) Show that $\chi(G) = 3$ for a Peterson Graph(G). Devise a method to color the vertices. (10)