B. E. Computer Science and Engineering Examination, 2018

(1st Year, 1st Semester, Supplemantary)

Mathematics- II

Time : Three hours Full Marks : 100

(Answer ***any five*** questions)

1. a) Prove that every convergent sequence is bounded. 10

   b) Prove that the sequence $\{x_n\}$ where $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$ is a convergent sequence. Estimate the value of the limit.

2. a) State Cauchy's general principle of convergence of an infinite series.

   Prove that if $u_n > 0$ and if

   $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = \rho$ then

   i) Σu_n converges if $\rho > 1$

   ii) diverges if $\rho > 1$

   b) Test the convergence of the following series :

   $$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \cdots$$ 10

3. a) Show that the function

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & x^2+y^2\neq 0 \\ 0, & x=0=y \end{cases}$$

is continuous at (0, 0) 10

b) If $u=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin 2u \qquad 10$$

4. a) State and prove Leibnitz's theorem of nth derivative of the product of two functions. 10

b) Find the value of y_n for $x=0$ when $y=e^{a\sin^{-1}x}$ 10

5. a) Prove that a continuous bounded function on [a, b] is integrable on [a, b] 10

b) Prove that the function f(x) defined as

$$f(x)=\begin{cases} x, & \text{when } x \text{ is rational} \\ -x, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable over [a, b], but $|f|$ is integrable. 10

6. a) Test the convergency of the following

i) $\int_0^{\pi/2} \log \sin x\, dx$

ii) $\int_0^{\infty} \frac{\sin x}{x} dx$ 6+4

b) Evaluate :

i) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

ii) $\lim_{x\to 0}\left(\frac{x-\sin x}{x^3}\right)$ 10

7. a) Show that in the cycloid $x=a(\theta+\sin\theta)$ $y=a(1-\cos\theta)$, $\rho^2+S^2=16a^2$ where the are 's' being measured from the verex $(\theta=0)$ and ρ is the radius f curvature at any point. 10

b) Find the area common to the cardiode $r=a(1+\cos\theta)$ and the circle $r=\frac{3}{2}a$. 10