Ex/CSE/Math/T/113A/2018(S)

B. E. Computer Science and Engineering Examination, 2018

(1st Year, 1st Semester, Supplemantary)

Mathematics- I

Time : Three hours Full Marks : 100

The figures in the margin indicate full marks

Answer **Q. No.9** and any six from **Q. No.1-8**.

1. (a) Let A, B,C, D be subsets of a set X. Prove that

$$(A\times C)\setminus(B\times D)=\{(A\setminus B)\times(C\setminus D)\}\cup \{(A\cap B)\times(C\setminus D)\}\cup\{(A\setminus B)\times(C\cap D)\}.$$ 8

(b) Define an equivalence relation ρ on a non-empty set S. Examine whether ρ is an equivalence relation on S in the following cases :

(i) S= $\mathbb{Z}\times\mathbb{Z}$ and (a,b) ρ (c,d) $\Leftrightarrow$ a+b = b+c.

(ii) S=($\mathbb{Z}\times\mathbb{Z}$) \{(0,0)} and (a,b) ρ (c,d) $\Leftrightarrow$ ad = bc. 8

2. (a) When is a function called left invertible ? Let A, B be two non-empty sets and $f: A \longrightarrow B$ be a function from A into B. Show that f is left invertible if and only f is injective.

(b) Let β be a permutation on the set $\{1,2,....7\}$ such that

$$\beta_4=\begin{pmatrix}1&2&3&4&5&6&7\\4&1&5&3&6&7&2\end{pmatrix}$$ then find β. 8+8

[Turn over

3. (a) Define a countable set. Prove that the every subset of the set $\mathbb{Q}$ of all rational number is countable. 8

   (b) What is a *cordinal number* ? Prove that the set of all real function defined on the closed unit interval has the cardinal number 2^c, Where c is the cardinal number of the set $\mathbb{R}$ of all real numbers. 8

4. (a) Find the truth table of $\left[(p \to q) \wedge (q \to r)\right] \to (p \to r)$

   . 8

   (b) Let A = $\{1,2,.....10\}$. Condiser each of the following sentences. If it is a statement, then determine its truth value. If it a proposional function, determine its truth set.

   (i) $(\forall x \in A)(\exists y \in A)(x + y < 14)$. 4

   (ii) $(\forall x \in A)(\forall y \in A)(x + y < 14)$. 4

5. (a) Prove that the diagonals of a parallelogram bisect each other, by vector method. 8

   (b) If $\vec{\alpha} = 2\vec{i} - 10\vec{j} + 2\vec{k}$, $\vec{\beta} = 3\vec{i} + \vec{j} + 2\vec{k}$. and $\vec{\gamma} = 2\vec{i} + \vec{j} + 3\vec{k}$.

   Find the vector $\vec{\alpha} \times (\vec{\beta} \times \vec{\gamma})$ and interpret the result geometrically. 8

6. (a) Show that the points A = (1, –2, 3), B = (2, –3, 4) and C= (–2, 1, 0) are collinear, by vector method. 8

   (b) Show that the vectors $2\vec{i} - \vec{j} + \vec{k}, \vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 4\vec{j} + 4\vec{k}$, from the sides of a right angled triangle. 8

7. (a) Find the equation of the cone whose vertex is at (1,2,3) and the guiding, curve is the circle

   $x^2 + y^2 + z^2 = 0$, $x + y + z = 1$

   (b) Find the equation of the cylinder whose gererating line is parallel to the z-axis and the guiding curve is given by , $x^2 + y^2 - z = 0$, $x + y + z = 1$ 8+8

8. (a) If the volume of a tetrahedron be 2 units and three of its vertices be (1,1,0), (1,0,1) and (2, –1, 1), then find the locus of the fourth vertex. 8

   (b) Find the torque about the point (3, –1, 3) of a force (4i+2j+k) passing through the point (5, 2,4). 8

9. What is the remainder when $1! + 2! + 3! + ... + 99! + 100!$ is divided by 18 ? 4