7. a) Evaluate :

i) $\operatorname*{Lt}_{x\to 0} (\cos x)^{\cot^2 x}$

ii) $\operatorname*{Lt}_{x\to 0} \dfrac{x-\sin x}{x^3}$

b) Find the perimeter of the cardioide $r = a(1-\cos\theta)$, and show that the arc of the upper half of the curve is bisected by $\theta = \dfrac{2\pi}{3}$. 7

c) Find the asymptotes of the curve

$(y-2x)^2(y-x)-3(y-2x)(y-x)+2(y-x)+1=0$ 7

Ex/CSE/Math/T/114A/2018

# Bachelor of Engineering in Computer Science Engineering Examination, 2018

( 1st Year, 1st Semester )

## Mathematics II

Time : Three hours Full Marks : 100

Answer ***any five*** questions.

All symbols and notations have their usual meanings.

1. a) Show that the sequence $\{x_n\}_{n\in N}$, where $x_n = \dfrac{1}{n+1}+\dfrac{1}{n+2}+\cdots+\dfrac{1}{2n}$ is monotonically increasing bounded. 8

b) Check whether the sequence $\{2^n\}_{n\in N}$ is a cauchy sequence or not. 4

c) Define limit of a sequence. Prove that a convergent sequence determines its limit uniquely. 8

2. a) Test the convergence of the series

$$\frac{1+2}{2^3}+\frac{1+2+3}{3^3}+\frac{1+2+3+4}{4^3}+\cdots$$ 8

b) Find the radius of convergence, and the exact interval of convergence of the power series.

$$1-(x-1)+\frac{(x-1)^2}{2}-\frac{(x-1)^3}{3}+\cdots$$ 6

c) Test the convergence of the series

$$\sum_{n=2}^{\infty} \frac{\log n}{\sqrt{n+1}}. \qquad 6$$

3. a) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} y \sin \frac{1}{x} + \frac{xy}{x^2 + y^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Verify that $\lim_{x \to 0} \lim_{y \to 0} f(x, y)$ exists but neither $\lim_{y \to 0} \lim_{x \to 0} f(x, y)$ nor $\lim_{(x,y) \to (0,0)} f(x, y)$ exists. 8

b) Prove that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x, y) = \sqrt{|xy|}$ is not differentiable at the origin, but $f_x$ and $f_y$ both exist at the origin. 8

c) If $z = e^{xy^2}, x = t \cos t, y = t \sin t,$ compute $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$. 4

4. a) State Taylor's theorem for a function of two variables. Apply it to find the expansion of $f(x, y) = x^2 y + 3y - 2$ in powr of $(x - 1)$ and $(y + 2)$. 8

b) Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. 6



c) If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, prove that

$$\left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}\right)^2 u = -\frac{\sin u \cos 2u}{4 \cos^3 u} \qquad 6$$

5. a) State and prove Leibnitz's theorem of nth derivative of the product of two function . 8

b) Find the value of $y_n$ for $x = 0$ when

$$y = e^{a \operatorname{Sin}^{-1} x} \qquad 8$$

c) Show that

$$\frac{x}{1 + x} < \log(1 + x) < x, \text{ if } x < 0. \qquad 4$$

6. a) Prove that a bounded function f(x), having a finite number of points of discontinuity on [a, b] in integrable on [a, b]. 8

b) Compute the value of the integral $\int_0^1 x^2 dx$ by Riemann integral theory. 4

c) Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)}$. 8