Bachelor of Computer Science & Engineering Examination 2018

(First Year, First Semester)

MATHEMATICS - I

Time : Three Hours Full Marks : 100

The figures in the margin indicate full marks

Answer Q. No. 9 and any **six** questions from Q. Nos. 1 – 8.

1. Let A, B, C be three subsets of a set X.

   (a) Show that $A\Delta(B\Delta C) = (A\Delta B)\Delta C$. 8

   (b) If $A\Delta C = B\Delta C$, then prove that $A = B$. 8

2. (a) If $\beta = (1\ 2\ 3\ 7)(1\ 4\ 5\ 6) \in S_7$, write β^{-99} in disjoint cycle notation. 8

   (b) Let $\beta = (1\ 3\ 5\ 7\ 9\ 8\ 6)(2\ 4\ 10) \in S_{10}$. What is the smallest positive integer n for which $\beta^n = \beta^{-5}$? 8

3. (a) Define a *countable* set. Let X and Y be non-empty sets and $f : X \longrightarrow Y$ be a surjective map. If X is countable, then prove that Y is also countable. 8

   (b) Define the *cardinal number* of a set. Prove that the set of all real functions defined on the closed unit interval has cardinal number 2^c. 8

4. (a) Prove logically that, "If p implies q and q implies r, then p implies r." 8

   (b) Determine the validity of the argument: $p \to \neg q,\ r \to q, r \vdash \neg p$. 8

5. (a) Show that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$. Hence prove that $\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = \vec{0}$. 8

   (b) Show that the straight lines whose direction cosines are given by $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and they are parallel if $\sqrt{af} + e_1\sqrt{bg} + e_2\sqrt{ch} = 0$, where $e_1, e_2 \in \{1, -1\}$. 8

6. (a) A variable plane makes intercepts on the co-ordinate axes, the sum of whose squares is constant and equal to k^2. Show that the locus of the foot of perpendicular from the origin to the plane is $(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2})(x^2 + y^2 + z^2)^2 = k^2$. 8

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(b) Find the equation of the straight line $x - 2y + 3z = 4$, $2x - 3y + 4z = 5$ in symmetrical form and find its direction cosines. 8

7. (a) Show that the shortest distance between the lines $y = az + b$, $z = \alpha x + \beta$ and $y = a'x + b'$, $z = \alpha' x + \beta'$ is

$$\frac{(\alpha - \alpha')(b - b') - (\alpha'\beta - \alpha\beta')(a - a')}{\{\alpha^2\alpha'^2(a - a')^2 + (\alpha - \alpha')^2 + (a\alpha - a'\alpha')^2\}^{\frac{1}{2}}}.$$

8

(b) Find the volume of tetrahedron in terms of co-ordinates of the vertices. Hence find the volume if one vertex is at the origin. 8

8. (a) Prove that the equation to a sphere circumscribing the tetrahedron whose faces are $\frac{y}{b} + \frac{z}{c} = 0$, $\frac{z}{c} + \frac{x}{a} = 0$, $\frac{x}{a} + \frac{y}{b} = 0$, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is $\frac{x^2+y^2+z^2}{a^2+b^2+c^2} - \frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$. 8

(b) Find the equation of a right circular cone whose vertex is at the point (α, β, γ), the semi-vertical angle is θ and the axis is $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$. Using this find the equation of the cone, whose vertex is at the origin and base is the circle $x = a$, $y^2 + z^2 = b^2$. 8

9. Let $S = \{n \in \mathbb{N} \mid n \text{ divides } 144\}$. Define a *lattice*. Define a partial order $\leqslant$ on S such that $(S, \leqslant)$ is a lattice. 4