7. a) What is an ideal of a ring ? Define a principal ideal. 3

b) Consider the ring of all integers. For two positive integers a, b show that

$$(a)+(b)=(d) \qquad d=\gcd(a,b)$$

$$(a)\cdot(b)=(c) \qquad c=\text{Lcm}(a,b)$$

where (x) is the principal ideal generated by x. 8

c) Show that in a commutative ring R with identity, a proper ideal P of R is prime iff R/P is an integral domain. 9

8. a) Prove that a polynomial of degree n over a field F can have at most n roots in any extension field. 7

b) If p is a prime, prove that $(Zp, +\cdot)$ is a field. 7

c) Show that in a commutative ring R with identity, an ideal of R is maximal iff R/M is a field. 6

Ex/CSE/Math/T/125/2018 (Old)

# Bachelor of Engineering in Computer Science and Engineering Examination, 2018

( 1st Year, 2nd Semester )

## Mathematics - IVD

Time : Three hours Full Marks : 100

( Answer ***any five*** questions )

1. a) Find the rank of the matrix

$$\begin{pmatrix} 2 & 0 & 2 & 2 \\ 3 & 4 & -1 & -9 \\ 1 & 2 & 3 & 7 \\ -3 & 1 & -2 & 0 \end{pmatrix}$$ reducing it to normal form. 8

b) Solve the following sytem of equations (if possible)

i)
$$\begin{aligned} x+y+z+w&=0 \\ 3x+4y-z&=0 \\ x+2y-3z+u&=0 \end{aligned}$$

ii)
$$\begin{aligned} x-2y+z-w&=-1 \\ 3x-2z+2w&=-4 \\ 5x-4y+w&=-5 \end{aligned}$$
6+3

c) Explain with reasons whether

$$(A+B)^2 = A^2+2AB+B^2$$

hold for square matrices. 3

[ Turn over

2. a) Find the eigen values and eigen vectors of the matrix

$$\begin{pmatrix} 3 & -2 & 1 \\ 2 & -1 & 0 \\ -2 & 2 & 0 \end{pmatrix}$$ 9

b) State and prove Caley-Hamilton Theorem. 6

c) Define a vector space. 2

3. a) In a vector space $(\vee, +, \cdot)$ prove that

i) $(-c)\alpha = c(-\alpha) = -(c\alpha)$

ii) $c\alpha = \theta \Rightarrow c = 0$ or $\alpha = \theta \;\; \forall \alpha \in \vee, \;\; c \in F.$ 6

b) Define the direct sum of two subspaces of a vector space. If M and N are two subspaces of a vector space $(\vee, +, \cdot)$ then prove that $V = M \oplus N$ 1+6

c) Show that (1, 1, 1), (1, 2, 3), (2, –1, –1) forms a basis of $R^3$ and express (3, 4, 5) as a linear combination of these vectors. 7

4. a) Define the basis of a vector space. Prove that any finite dimensional vector space has a basis. 2+6

b) If $\{\alpha, \beta, \gamma\}$ is a basis of a vector space V then show that $\{\alpha + \beta, \;\beta + \gamma, \;\gamma + \alpha\}$ is also a basis of V. 4



c) Find the basis and dimension of

i) $S = \{(x, y, z) \in R^3 : 2x - y + 3z = 0\}$

ii) S is the space of all 3rd order symmetric matrices.

8

5. a) Define a linearly dependent set of vectors. Prove that a set of r non-zero vectors $\{\alpha_1, \alpha_2, \cdots, \alpha_r\}$ is linearly dependent iff. at least one of $\alpha_i$ is a linear combination of preceeding vectors. 1+7

b) Define a group. Prove that $(G, \cdot)$ is a group iff for any two elements a, b $\in$ G we have $a^{-1}b \in G$. 1+5

c) Show that a finite group $(G, \cdot)$ is a cyclic group iff there exists an element $a \in G$ such that $0(a) = |G|$. 6

6. a) Let H be a subgroup of a finite group G. Show that

i) $|aH| = |H| = |Ha| \;\; \forall a \in G$

ii) O(H) divides O(G). 10

b) If H and K are two subgroups of a group G then show that HK is a subgroup of G iff $HK = KH$. 7

c) Define a ring. Give an example o a commutative ring. 3