BACHELOR OF ENGINEERING IN CIVIL ENGINEERING (EVENING) EXAMINATION, 2018 (OLD)
(1st Year, 2nd Semester)
MATHEMATICS - II

Time : Three hours

Full Marks : 100

Answer any *six* questions.
Four marks are reserved for neatness.
(Notations have their usual meanings)

1 a) If $\vec{r} = (a\cos t)\vec{i} + (a\sin t)\vec{j} + (a\,t\tan\alpha)\vec{k}$, then show that
$$\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dr^2}\ \frac{d^3\vec{r}}{dr^3}\right] = a\tan\alpha$$
b) Show that $[\,\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\,] = 2[\,\vec{a}\,\vec{b}\,\vec{c}\,]$
c) In any triangle ABC, with usual notations, prove that a = bcosC + ccosB. 6+5+5

2 a) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point (2,-1, 1) in the direction of the normal to the surface $xlogz - y^2 \doteq -4$ at (-1, 2, 1).
b) Prove that

$$\vec{\nabla}(\vec{A}.\vec{B}) = (\vec{B}.\vec{\nabla})\vec{A} + (\vec{A}.\vec{\nabla})\vec{B} + \vec{B}\,X(\vec{\nabla}X\vec{A}) + \vec{A}\,X(\vec{\nabla}X\vec{B})$$ 8+8

3 a) Find the value of the constant d such that the vecors (2i – j + k), (1 + 2j – 3k) and (3i + dj + 5k) are coplanar.

b) If a X b = c, b X c = a and c X a = b, then show that

(a X b) X (c X d) = 0.

c) Prove that,

(α X β).(γ X δ) + (α X γ).(δ Xβ) + (α X δ).(β X γ) = 0. 5+5+6

4 a) Find the directional derivative of $\varphi = xy^2z + 4x^2z$ ath the point (-1, 1, 2) in the direction of the vector 2I + J – 2K.

b) Show that the vectors $F = 3y^4z^2\mathbf{i} + 4x^3z^2\mathbf{j} - 3x^2y^2\mathbf{k}$ is solenoidal.

c) Show that the vector (ysinz - sinx)**i** + (xsinz + 2yz)**j** + (xy cosz + y^2) **k** is irrotational. 6+5+5

5. a) Let $u = x_1x_2 + x_2x_3 + x_3x_1$. Find the relative percentage error in computing u at $x_1 = 2.104$, $x_2 = 1.935$ and $x_3 = 0.845$.

b) From the interpolation polynomial for the function y = f(x) are given by the table. Find f(x) and hence deduce f(1).

x	-1	1	4	6
f(x)	1	-3	21	127

c) Find the real root of $x^3 - 3x - 5 = 0$ by the method of False position, correct upto five significant figures.

5+6+5

6. a) Use suitable interpolation formulae to compute f (0.23) and f(0.29) from the following data:

x	0.20	0.22	0.24	0.26	0.28	0.30
f(x)	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

b) Calculate by Simpson's one-third rule the value of the integral

$$\int_0^6 x^2(1-x)dx,$$

Correct upto three places of decimals by taking step-length eaual to 0.1.

10+6

7. a) Evaluate

$$\int_0^1 (4x - 3x^2)\, dx$$

by Trapezoidal rule with ten subintervals and compare the results.

b) Use Newton-Raphson method to evaluate the real root of the equations $x^3 + 2x - 2=0$, correct to three significant figures.

c) Solve the equation $x^3 - 9x + 1=0$ for the roots lying between 2 and 4.

6+5+5

8a) A bag contains 4 white and 6 black balls. Two balls aresuccessively drawn from the urn without replacement of the first ball. If the first ball is seen to be white, what is the probability that the 2nd ball is also white.

b) A random variable X has the following probability function :

x	-2	-1	0	1	2	3
f(x)	0.1	K	0.2	2K	0.3	3K

i) Find K,

ii) Evaluate $P(X < 2)$, $P(X \leq 2)$, $P(-2 < X < 2)$.

iii) Determine the distribution function F(x) of X.

6+10