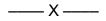
(b) $\sum \frac{1}{(2n-1)^2} = \frac{n^2}{8}$

12. Obtain the half range cosine and sine series for f(x) = x in the interval $0 \le x \le \pi$.



BACHELOR OF POWER ENGINEERING EXAMINATION, 2019 (1st Year, 1st Semester)

Mathematics - II Q

Time: Three hours Full Marks: 100

Notations/Symbols have their usual meaning

Answer any *ten* questions.

1. (a) Find the real values of x, y so that $-3 + ix^2y$ and $x^2 + y + 4i$ may represent complex conjugate numbers.

(b) Simplify:
$$\frac{\left(\cos 3\theta + i\sin 3\theta\right)^4 \left(\cos 4\theta - i\sin 4\theta\right)^5}{\left(\cos 4\theta + i\sin 4\theta\right)^3 \left(\cos 5\theta + i\sin 5\theta\right)^{-4}}$$

(c) If
$$2\cos\theta = x + \frac{1}{x}$$
, prove that
$$2\cos n x = x^n + \frac{1}{x^n}$$
 3+4+3

2. (a) Show that

$$\sin^{8}\theta = \frac{1}{2^{7}}(\cos 8\theta - 8\cos 6\theta + 28\cos 4\theta - 56\cos 2\theta + 35)$$

(Turn over)

(b) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)$$

5+5

- 3. (a) If a+b+c=0, solve $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$
 - (b) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix:

$$\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$$

(c) Show that the reciprocal of the product of two matrices is the product of their reciprocals taken in the reverse order.

4+3+3

9. Obtain the various possible solutions of the onedimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \,,$$

by the mathod of separation of variables. Hence identify the most appropriate solution (with justification).

10. Obtain the solution of the one-dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} , 0 \le x \le L, t > 0$$

satisfying the conditions:

$$u(0,t) = 0$$
 & $u(L,t) = 0$ for $t > 0$ and $u(x,o) = f(x)$, $0 \le x \le L$.

11. Find the Fourier series expansion of $f(x) = x^2$ in the interval $-\pi \le x \le \pi$. Hence deduce the following results:

(a)
$$\sum \frac{1}{n^2} = \frac{n^2}{6}$$

(Turn over)

(b) Test the convergence of the series

$$\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$$
 6+4

- 7. (a) Form a PDE by eliminating the arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = c^2$.
 - (b) Form a PDE by eliminating the arbitrary function f from $f(x^2 + y^2, z xy) = 0$
 - (c) Solve: $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when x = 0, $z = e^y$

and
$$\frac{\partial z}{\partial x} = 1$$
. $3+4+3$

- 8. Solve the following PDEs:
 - (a) (z-y)p + (x-z)q = y-x
 - (b) $xp yq = y^2 x^2$

(c)
$$x(y-z)p + y(z-x)q = z(x-y)$$
 3+4+3

4. (a)

If
$$A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}$$
,

find adj A and A^{-1} verify that $AA^{-1} = I$.

- (b) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$.
- 5. (a) Solve the following equations by Cramer's rule : x+3y+6z=2, 3x-y+4z=9, x-4y+2z=7
 - (b) Solve the following system of equations by the matrix method:

$$x+y+z=3$$
, $x+2y+3z=4$, $x+4y+9z=6$. 5+5

- 6. (a) Show that the series $1+r+r^2+r^3+....\infty$
 - (i) converges if |r| < 1,
 - (ii) diverges if $r \ge 1$, and
 - (iii) oscillates if $r \le -1$.

(Turn over)