(b) $\sum \frac{1}{(2n-1)^2} = \frac{n^2}{8}$ 10

12. Obtain the half range cosine and sine series for f(x) = x in the interval $0 \le x \le \pi$. 10

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Ex./PE/MATH/T/113/2019(OLD)

BACHELOR OF POWER ENGINEERING EXAMINATION, 2019

(1st Year, 1st Semester)

Mathematics - II Q

Time : Three hours Full Marks : 100

Notations/Symbols have their usual meaning

Answer any ***ten*** questions.

1. (a) Find the real values of x, y so that $-3 + ix^2y$ and $x^2 + y + 4i$ may represent complex conjugate numbers.

   (b) Simplify : $\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$

   (c) If $2\cos\theta = x + \frac{1}{x}$, prove that

   $2\cos nx = x^n + \frac{1}{x^n}$ 3+4+3

2. (a) Show that

$$\sin^8\theta = \frac{1}{2^7}(\cos 8\theta - 8\cos 6\theta + 28\cos 4\theta - 56\cos 2\theta + 35)$$

(b) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

5+5

3. (a) If $a+b+c=0$, *solve* $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

(b) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix :

$$\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$$

(c) Show that the reciprocal of the product of two matrices is the product of their reciprocals taken in the reverse order. 4+3+3

9. Obtain the various possible solutions of the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} ,$$

by the mathod of separation of variables. Hence identify the most appropriate solution (with justification). 10

10. Obtain the solution of the one-dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \ , 0 \le x \le L, t > 0$$

satisfying the conditions :

u(0,t) = 0 & u(L,t) = 0 for t > 0 and u(x,o) = f(x), $0 \le x \le L$. 10

11. Find the Fourier series expansion of $f(x) = x^2$ in the interval $-\pi \le x \le \pi$. Hence deduce the following results :

(a) $\sum \frac{1}{n^2} = \frac{n^2}{6}$

(Turn over)

(b) Test the convergence of the series

$$\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$ 6+4

7. (a) Form a PDE by eliminating the arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = c^2$.

(b) Form a PDE by eliminating the arbitrary function f from $f(x^2 + y^2, z - xy) = 0$

(c) Solve : $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. 3+4+3

8. Solve the following PDEs :

(a) $(z-y)p + (x-z)q = y-x$

(b) $xp - yq = y^2 - x^2$

(c) $x(y-z)p + y(z-x)q = z(x-y)$ 3+4+3

4. (a)

If $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}$,

find adj A and A^{-1} verify that $AA^{-1} = I$.

(b) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$. 7+3

5. (a) Solve the following equations by Cramer's rule :

$x + 3y + 6z = 2$, $3x - y + 4z = 9$, $x - 4y + 2z = 7$

(b) Solve the following system of equations by the matrix method :

$x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$. 5+5

6. (a) Show that the series $1 + r + r^2 + r^3 + \ldots\ldots\infty$

(i) converges if $|r| < 1$,

(ii) diverges if $r \geq 1$, and

(iii) oscillates if $r \leq -1$.