11. (a) If $\vec{\alpha}\times\left(\vec{\beta}\times\vec{\gamma}\right)=\frac{1}{2}\vec{\beta}$, find the angles which $\vec{\alpha}$ make with $\vec{\beta}$ and $\vec{\gamma}$, where $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are unit vectors. 4

(b) If xy = 9, find the maximum and minimum value of 9x + 4y. 6

12. (a) Verify Rolle's theorem for f(x) = Sin x + Cos x–1 in $[0, \pi/2]$. 5

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$$ 5

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Ex./PE/MATH/T/112/2019(OLD)

**BACHELOR OF POWER ENGINEERING EXAMINATION, 2019**

**(1st Year, 1st Semester, Old Syllabus)**

**Mathematics - I Q**

Time : Three hours Full Marks : 100

Answer any ***ten*** questions.

1. (a) Prove that Kronecker delta is a tensor of type (1,1). 4

(b) If a metric in a space $V^2$ is given by $ds^2 = (dx^1)^2 + 3dx^1dx^2 + 5(dx^2)^2$, find $g_{ij}$ and $g^{ij}$. What type of tensors are $g_{ij}$ and $g^{ij}$? 6

2. (a) If (2,3) are components of a contravariant vector in $(x^i)$ coordinate system, then find its components in $\left(\bar{x}^i\right)$ coordinate system, where

$$\bar{x}^1 = 2x^1 - 3x^2, \bar{x}^2 = -5x^1 + 2x^2.$$

(b) Find the value of $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]$. 5+5

3. (a) Prove by vector method

Cos(A+B) = Cos A Cos B – Sin A Sin B. 6

(b) Find a unit vector perpendicular to each of the vectors $\vec{p}+\vec{q}$ and $\vec{p}-\vec{q}$, where $\vec{p}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{q}=\hat{i}-2\hat{j}+2\hat{k}$. 4

4. (a) Evaluate $\int_{-1}^{1}\frac{dx}{x^2}$ if possible. What is the Cauchy value of this integration? 5

(b) Find (i) $\int_{-\infty}^{\infty}\frac{dx}{4x^2+16}$ (ii) $\int_{0}^{2}\frac{dx}{\sqrt{4-x^2}}$ 5

5. (a) Find $\int_0^1\int_0^1\int_0^1 xyz\,dx\,dy\,dz$. 5

(b) Evaluate $\iint_R xy\left(x^2+y^2\right)dxdy$ in the region R : {[0,2], [0,3]}. 5

6. (a) Find $\int_0^{\pi/2} Sin^5\theta\, Cos^7\theta\, d\theta$ by using Beta-Gamma function. 5

(b) Find $\Gamma\left(\frac{1}{2}\right)$ *and* $\Gamma(7)$. 5



7. (a) State Rolle's theorem and explain it geometrically. 4

(b) Verify Lagrange's Mean value theorem for the function f(x) = (x–1)(x–2)(x–3) in [0,4]. 6

8. (a) If $y = \text{Cos}(m\,\text{Sin}^{-1}x)$, then prove that

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$$ 5

(b) If $y=\frac{x^3}{x^2-1}$ prove that $(y_n)_0 = 0$, if n is even. 5

9. (a) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$. 5

(b) Find the expansions of the function f(x) = Sin x in infinite series. 5

10. (a) State Euler's theorem on homogeneous function of two variables. Hence use it to prove if

$$u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right) \text{ then } x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u.$$ 6

(b) If $z = f(x+ct) + \phi(x-ct)$, then prove that

$$\frac{\partial^2 z}{\partial t^2}=c^2\frac{\partial^2 z}{\partial x^2}.$$ 4

(Turn over)