(b) Show that the straight lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Find the point of intersection also. 5+5

11. (a) Find the equation of a sphere which passes through origin and intercepts lengths a, b and c on the axes respectively.

(b) Find the equation of the sphere which passes through the circle

$$x^2+y^2+z^2=25,\ x+2y+3z=6$$

and whose centre lines on the plane $2x+4y-3z=2$. 5+5

12. (a) Find the solution of the following wave equation described by

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$$

subject to boundary conditions u(0,t) = 0, u(5, t) = 0

and initial conditions $u(x,0)=0, \left(\frac{\partial u}{\partial t}\right)_{t=0}=5\sin \pi x$.

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Ex:/PE/MATH/T/122/2019(OLD)

BACHELOR OF POWER ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - III Q

Time : Three hours Full Marks : 100

Symbols/Notations have there usual meanings.

Answer any ***ten*** questions.

1. (a) Find the curvature and torsion for a space curve $\vec{r}=a\cos t\,\hat{i}+a\sin t\,\hat{j}+bt\,\hat{k}$ at any point of it.

(b) If $\vec{R}=x^2yz\hat{i}+2xyz\hat{j}+yz^2\hat{k}$, *find* $\frac{\partial\vec{R}}{\partial x}, \frac{\partial\vec{R}}{\partial y}$ & $\frac{\partial\vec{R}}{\partial z}$. 6+4

2. (a) If $\vec{V}=(x+3y)\hat{i}+(y-2z)\hat{j}+(9-2z)\hat{k}$, check whether vector field $\vec{V}$ is solenoidal or irrotational ?

(b) If $\varphi=x^2y^3z^4$, find directional derivative of φ at (2,1,1) in the direction of $3\hat{i}+6\hat{j}+2\hat{k}$. 5+5

3. (a) If $\vec{a}=t\hat{i}-t^2\hat{j}+(t-1)\hat{k}$ and $\vec{b}=2t^2\hat{i}+6t\hat{k}$ evaluate $\int_0^2(\vec{a}\cdot\vec{b})dt$ *and* $\int_0^2(\vec{a}\times\vec{b})dt$.

(b) Solve the differential equation $y = px + {}^{q}/_{p}$, where $p = \dfrac{dy}{dx}$ and interpret the results. 2+3+5

4. Verify Stokes' theorem for $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. 10

5. Solve the following differential equations :

(i) (x+y+1)dx – (2x+2y+1)dy = 0

(ii) $\left(1+x^2\right)\dfrac{dy}{dx} + y = e^{\tan^{-1}x}$ 5+5

6. Solve :

(a) $(D^2+4)y = \sin 3x$

(b) $(D^2 - 4D + 3)y = e^{3x}$ 5+5

7. (a) If the curves $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$ *and* $\dfrac{x^2}{c} + \dfrac{y^2}{d} = 1$ intersect at right angles, then prove that a – b = c – d.

(b) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at any point makes an angle φ with the positive direction of the x-axis, then prove that, the equation of the normal is y cosφ – x sinφ = a cos 2φ. 5+5

8. (a) Prove that the acute angle between two diagonals of a cube is $\text{Cos}^{-1}\left(\dfrac{1}{3}\right)$.

(b) Prove that the two lines whose direction cosines are connected by the two relations al + bm + cn = 0 and $ul^2 + vm^2 + wn^2 = 0$ are parallel if $\dfrac{a^2}{u} + \dfrac{b^2}{v} + \dfrac{c^2}{\omega} = 0$. 5+5

9. (a) A variable plane is a constant distance p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16\,p^{-2}$.

(b) Find the equation of the plane which passes through the points (1,0,1) and (2,–1,1) and is parallel to the line $\dfrac{x-1}{2} = \dfrac{y-2}{-2} = \dfrac{z+1}{3}$. 5+5

10. (a) Find the shortest distance between the two straight lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x}{2} = \dfrac{y-5}{3} = \dfrac{z+1}{4}$.