Ex./CE/MATH/T/221/2018(OLD)

BACHELOR OF CIVIL ENGG. EXAMINATION, 2018

(2nd Year, 2nd Semester, Old Syllabus)

Mathematics - IV C

Time : Three hours | Full Marks : 100

Use a separate Answer Script for each part.

Symbols/Notations have their usual meanings.

PART - I (50 marks)

Answer any ***five*** questions.

1. (a) Show that the function $f(z)=\begin{cases}\dfrac{(\bar{z})^2}{z}, & z\neq 0\\ 0, & z=0\end{cases}$

satisfies Cauchy-Riemann equations at (0,0), but the function is not differentiable at origin.

(b) Show that $\lim_{z\to\infty}\frac{1}{z^2}=0$ 6+4

2. Define singular point. Also show that $u = x^3 - 3xy^2 - 3x^2 - 3y^2 + 1$ is a harmonic function and find the corresponding analytic function. 2+8

(Turn Over)

3. (a) A complex valued function f(z) is defined by

$$f(z)=\begin{cases}\dfrac{\operatorname{Im} g\, z}{|z|}, & \text{if } z\neq 0\\ 0, & \text{if } z=0\end{cases}$$

Is f(z) continuous of z = 0 ?

(b) Show that polar form of Cauchy Riemann equations are $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$ 5+5

4. (a) If $\vec{r}=a\cos t\,\hat{i}+a\sin t\,\hat{j}+bt\,\hat{k}$, then show that

$$\left|\frac{d\vec{r}}{d\vec{t}}\right|=a^2+b^2$$

(b) Find the directional derivative of f = xy + yz + zx in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$ at (1,2,0). 4+6

5. (a) Find the curvature and torsion for the curve x = a cos t, y = a sint, z = bt.

(b) A particle moving along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find components of its velocity and acceleration at time t=1, in the direction $\hat{i}-3\hat{j}+2\hat{k}$. 5+5

14. (a) Find the standard deviation of the following distribution :

x :	7	8	9	10	11	12	14
f :	2	3	4	5	3	2	1

(b) For a Binomial distribution the mean is 3 and $q=\frac{1}{2}$. Find n. 8+2

——X——

9. Find the median and mode for the following distribution. Hence find the mean. 10

Class:	25-29	30-34	35-39	40-44	45-49
Frequency:	16	28	14	12	7

10. Find the correlation coefficient of the two variables X and Y from the following data. Also find the regression equation of Y on X. 6+4

X :	5	6	7	8	10	12
Y :	15	18	24	26	27	32

11. Find f(2.5) using Newton's forward interpolation formula from the following table. 10

X :	1	2	3	4	5	6
Y :	0	1	10	81	256	625

12. Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ by using

(a) Trapezoidal rule

(b) Simpson $\frac{1}{3}$rd rule with $h = 0.5$ 5+5

13. State Newton-Raphson method to solve non-linear equations and compute a real roof of $f(x) = x - e^{-x} = 0$ using this method. 3+7

6. (a) Verify Stokes' theorem for

$\vec{F} = (2x+y)\hat{i} + yz^2\hat{j} + y^2 z\hat{k}$, when S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(b) Find the angle between

$\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ 7+3

7. (a) Suppose f(z) by an analytic function. Then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2$$

$$= 4|f'(z)|^2$$

(b) Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \text{grad }(x^3 + y^3 + z^3 - 3xyz)$. 5+5

PART - II (50 marks)

Answer any ***five*** questions.

8. (a) State and prove Baye's theorem.

(b) A dice is thrown three times in succession. Find the probability of getting two ones. 5+5

(Turn Over)