15. (a) Solve the one dimensional heat equation : 10

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 \le x \le l, \ t > 0,$$

satisfying the following conditions :

u(x,0) = 3 sin(nπx), u(0,t) = 0, u(l,t)=0

16. Solve the one dimensional wave equation :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \ 0 \le x \le l, \ t > 0,$$

together with following initial and boundary conditions :

u(x,0) = f(x), u_t(x,0) = g(x) for 0 < x < 1 and u(0,t) = u(l,t)=0 for t > 0. 10

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Ex./CE/MATH/T/121//2018

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - III C

Time : Three hours | Full Marks : 100

Use a separate Answer Script for each part.

Symbols/Notations have their usual meanings.

PART - I (50 marks)

Answer any ***five*** questions.

1. Solve :

(a) $xdy - ydx = \sqrt{x^2 + y^2}\, dx$

(b) $\left(D^2 + 4\right) y = \cos 2x \left(where \ D = \frac{dy}{dx} \right)$ 5+5

2. (a) Construct a differential equation by eliminating the parameter λ from the equation.

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$$

where a and b are fixed constants.

(b) Solve :

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2$$ 4+6

(Turn Over)

3. (a) Solve :

$(D^2–D–6)y = x$

(b) Find the singular solution of

$$p = \tan\left\{x - \frac{p}{1+p^2}\right\}, where\ p = \frac{dy}{dx}$$

OR

Solve :

$$\frac{dy}{dx} - \frac{3y}{x+2} = (x+2)^3$$ 5+5

4. (a) Find the Fourier series for the function.

$$f(x) = 0, -\pi < x \leq 0$$
$$= \frac{\pi x}{4}, 0 \leq x < \pi$$ Also find the value of $\frac{\pi^2}{8}$.

(b) Find the Fourier coefficients for the function

$$f(x) = 0 , 0 < x < 1$$
$$= 1, 1 < x < 2l.$$ 7+3

5. (a) Classify the singular points of the differential equation

$$(x^3 + x^2)y'' + (x^2 - 2x)y' + y = 0$$

(b) Prove that

$$\int_{-1}^{1} p_m(x)P_n(x)dx = \begin{cases} 0\ if\ m \neq mn \\ \frac{2}{2n+1}\ if\ m = n \end{cases}$$ 2+8

11. (a) Find the differential equation of all spheres, whose centres lie on the z-axis. 3

(b) Form the partial differential equation by eliminating the arbitrary function 'f' from the following : 4

$f(x+y+z, x^2+y^2+z^2) = 0$

(c) Solve the following partial differential equation : 3

$$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$$

12. Solve :

(a) $\frac{y^2 z}{x} p + xzq = y^2$

(b) $x(y–z)p + y(z–x)q = z(x–y)$ 5+5

13. Solve :

(a) $(p^2 + q^2)y = qz$

(b) $zpq = p + q$ 5+5

14. (a) Determine the solution of the Laplace's equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 z}{\partial x^2} = 0 , 0 < x < \pi, 0 < y < \pi$$

subject to the following initial and boundary conditions $u(0,y) = u(\pi,y) = u(x,\pi) = 0$, $u(x,0) = \sin^2 x$. 10

(Turn Over)

(c) Use Laplace transform method to solve the following differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t,\ x(0) = 0, x^{/}(0) = 1 \qquad 4$$

OR

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = 0,\ x(0) = x^{/}(0) = 1 \qquad 4$$

10. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not differentiable at $z = 0$, although Cauchy-Riemann equations are satisfied. 4

(b) If $f(z) = u(x,y) + iv(x,y)$ is an analytic function, find $f(z)$ if $u - v = e^x(\cos y - \sin y)$. 3

(c) Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f^{/}(z)| = 0$ 3

OR

Find the general solution of the following partial differential equations :

(a) $\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = xy$ 6

(b) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = 5e^{x+2y}$ 4

6. Find the series solution of Bessel's equation. 10

7. (a) Starting from generating function show that $(n+1)p_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

(b) Prove the following result for Bressel's function $J_n(x)$

$$\frac{d}{dx}\left\{x^n\ J_n(x)\right\} = x^n J_{n-1}(x) \qquad 5+5$$

8. Find the series solution near $x = 0$ of the differential equation 10

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 + 2\right)y = 0$$

Write at least three nonzero terms in each series.

PART - II (50 marks)

Answer any ***five*** questions.

9. (a) If $L\{f(t)\} = F(s)$, then find $L\{e^{at}f(t)\}$. 3

(b) Find the inverse Laplace transform of

$$\frac{5s+3}{(s-1)(s^2+2s+5)} \quad or \quad \frac{1}{\sqrt{2s+3}} \qquad 3$$