EX/CE/MATH/ T/113/2018

B.CIVIL ENGG. Examination, 2018

(1ST YR, 1ST SEM)

MATHEMATICS

PAPER - II C

Full Marks : 100 Time: Three hours

(use separate answer script)

Part - I

Answer any five questions in each part.

$(10 \times 10 = 100)$

1. (a) Find the eigen values and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) If λ be an eigen value of a non singular martix A , then prove that λ^{-1} is an eigen value of A^{-1}

2. (a) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2\,a\,b\,c\,(a+b+c)^3$$

(b) If A is real orthogonal matrix and $(I+A)$ is non singular then prove that $(I+A)^{-1}(I-A)$ is skew symmetric matrix.

3. (a) State Cayley-Hamilton theorem. Verify Cayley-Hamilton theore for the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{bmatrix}$$

(b) Solve by inverse method

$$2x - 5y + 6z = 9$$
$$x - y + 3z = 6$$
$$2x + 3y - z = -4$$

4. (a) If $x + \frac{1}{x} = 2\cos\frac{\pi}{7}$, then show that $x^7 + \frac{1}{x^7} = -2$.
(b) If $a + \frac{1}{a} = 2\cos\alpha$, then prove that

$$a^n + \frac{1}{a^n} = 2\cos n\alpha \quad and \quad a^n - \frac{1}{a^n} = 2i\sin n\alpha.$$

5. Test for convergence of the following series.

$$(i)\ x + \frac{1}{2}.\frac{x^3}{3} + \frac{1.3}{2.4}.\frac{x^5}{5} + \ldots\ldots$$

$$(ii)\ \left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \ldots\ldots$$

$$(iii)\ \frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \ldots\ldots$$

6. (a) Solve by Cramer's rule

$$ax+by+cz = 1, \quad cx+ay+bz = 0, \quad bx+cy+az = 0$$

(b) Expand θ in powers of $\tan\theta$.

Part - II

7. (a) Show that

$$|\vec{A} \times \vec{B}|^2 + |\vec{A}.\vec{B}|^2 = |\vec{A}|^2|\vec{B}|^2$$

(b) Prove the law of sines for plane triangle.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

8. (a) Find the area of the triangle having the vertices at $\vec{P}\,(1,2,3)$, $\vec{Q}\,(2,-1,1)$,$\vec{R}\,(-1,2,3)$

(b) Find the unit vector parallel to the xy plane and perpendicular to the vector $4\hat{\imath} - 3\hat{\jmath} + \hat{k}$

9. (a) Prove that

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)+\vec{B}\times\left(\vec{C}\times\vec{A}\right)+\vec{C}\times\left(\vec{A}\times\vec{B}\right)=0$$

(b) Prove that

$$\left(\vec{A}\times\vec{B}\right).\left(\vec{C}\times\vec{D}\right)=\left(\vec{A}.\vec{C}\right)\left(\vec{B}.\vec{D}\right)-\left(\vec{A}.\vec{D}\right)\left(\vec{B}.\vec{C}\right)$$

10. (a) A plane passing through a fixed point (a,b,c) cuts the axis in A , B , C show that the locus of the center of the sphere OABC is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2.$$

(b) Show that the unit vector perpendicular to both the vectors $\left(3\hat{\imath} + \hat{\jmath} + 2\hat{k}\right)$ and $\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right)$ is $\frac{1}{\sqrt{3}}\left(\hat{\imath} - \hat{\jmath} - \hat{k}\right)$ and the angle between them is $\sin^{-1}\frac{2}{\sqrt{7}}$.

11. (a) Find the center and the radius of the circle

$$x^2 + y^2 + z^2 = 25 \quad , \quad x + 2y + 2z + 9 = 0.$$

(b) Find the shortest distance between the straight lines

$$\frac{x-3}{-3} = \frac{y-8}{1} = \frac{z-3}{-1}$$

and

$$\frac{x+3}{3} = \frac{y+7}{-2} = \frac{z-6}{-4}$$

and the equations of the line of shortest distance.

12. Find the equations of straight lines in which the plane

$$2x + y - z = 0$$

cuts the cone

$$4x^2 - y^2 + 3z^2 = 0,$$

also find the angle between them.