BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2018

(1st Year, 2nd Semester)

Mathematics - II C (OLD)

Time : Three hours Full Marks : 100

Use a separate Answer Script for each part.

Symbols/Notations have their usual meanings.

PART - I (50 marks)

Answer q.no. 7 and any ***four*** from the rest.

1. (a) If a, z_1, z_2 are complex numbers such that $a \neq 0$, show that $a^{z_1} . a^{z_2} \neq a^{z_1+z_2}$. Is the principal value of $a^{z_1+z_2}$ equal to the product of the principal values of a^{z_1} and a^{z_2} ? Justify. 5

   (b) In a triangle ABC, show that $a^3 \cos 3B + 3a^2 b \cos(2B - A) + 3ab^2 \cos(B - 2A) + b^3 \cos 3A = c^3$, where A,B,C, a,b,c have their usual meanings. 5

2. (a) If x is real number, prove that

$$i\, log\left(\frac{x-i}{x+i}\right) = \begin{cases} \pi - 2\tan^{-1}(x)\ , \ if\ x > 0 \\ -\pi - 2\tan^{-1}(x) \ \ if\ x \le 0 \end{cases}$$ 5

(Turn Over)

(b) If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, $z = \cos\gamma + i\sin\gamma$ and $x+y+z = xyz$,
Prove that $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -1$. 5

3. (a) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $t^4 + t^2 + 1 = 0$ and 'n' is a positive integer then show that

$$\alpha^{2n} + \beta^{2n} + \gamma^{2n} + \delta^{2n} = 4\cos\left(\frac{2\pi n}{3}\right)$$ 5

(b) If z is a variable complex number such that an amplitude of $\frac{z-i}{z+1}$ is $\frac{\pi}{4}$, then show that the point z lies on a circle in the complex plane. 5

4. (a) State and prove Cauchy's root test for conveyence and / or divergence series of positive terms. 5

(b) Show that the series $\sum_{n-1}^{\infty} \frac{1}{np}$ is convergent of $p > 1$ and divergent of $p \le 1$ (not using integral test). 5

5. (a) Discuss the convergence of

$$2 + \frac{3}{8} + \frac{4}{27} + \ldots..$$ 5

12. (a) A plane parsing through a fixed point (a,b,c) cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$. 8

(b) Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$, $\frac{x+3}{3} = \frac{y+7}{2} = \frac{z-6}{4}$ and the equation of the line along which the distance is least. 8

13. (a) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in P, Q, R. Prove that the equation of the cone generated by lines drawn form 0 to meet the circle PQR is

$$yz\left(\frac{b}{c} + \frac{c}{a}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{c}\right) = 0$$ 8

(b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. 8

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9. (a) Show that

$$\begin{vmatrix} -bc & bc+b^2 & bc+c^2 \\ ca+a^2 & -ca & ca+c^2 \\ ab+a^2 & ab+b^2 & -ab \end{vmatrix} = (bc+ca+ab)^3$$ 10

(b) Prove that every skew symmetric determinant of third under is zero. 6

10. (a) Prove that every square matrix can be expressed as a sum of symmetric and a skew-symmetric matrix uniquely. 8

(b) If $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$

then show that matrix A is idempotent. 8

11. (a) Investigate, for what values of λ and μ, the following equations

$x+y+z=6$; $x+2y+3z=10$; $x+2y+\lambda z=\mu$

have (i) no solution (ii) an unique solution and (iii) an infinite number of solutions. 8

(b) Show that the system of equations $2y+4z+5=0$; $8x-y+4z=12$; $16x-y+10z=1$ is consistent. 8

(b) Find the region of convergence of the series

$$x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\ldots.$$ 5

6. (a) Prove that $\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$ by vector method. 5

(b) Show that the vectors $3\vec{a}-7\vec{b}-4\vec{c}$, $3\vec{a}-2\vec{b}+\vec{c}$, $\vec{a}+\vec{b}+2\vec{c}$ are coplanar, where $\vec{a},\vec{b},\vec{c}$ are any three non-coplanar vectors. 5

7. (a) Find the unit vector perpendicular to each of $\vec{a}=6\vec{i}+2\vec{j}+3\vec{k}$ and $\vec{b}=3\vec{i}-6\vec{j}-2\vec{k}$. 5

(b) Show by vector method the trigonometrical formula

$$\cos C = \frac{a^2+b^2-c^2}{2ab}.$$ 5

PART - II (50 marks)

Answer ***q.no. 8*** and any ***three*** from the rest.

8. Show that the orthogonal matrices are non-singular. 2