(b) If $a+\frac{1}{a}=2\cos\alpha$, then show that $a^n+\frac{1}{a^n}=2\cos n\alpha$,

$a^n-\frac{1}{a^n}=2i\sin n\alpha$ 5+5

10. (a) Express θ in terms of tan θ.

(b) If $\frac{z-i}{z+1}$, is purely imaginary then show that the point z lies on a circle. 5+5

11. (a) Test the convergence of the following series.

(i) $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}$ .....

(ii) $\frac{1}{1.3}+\frac{2}{3.5}+\frac{3}{5.7}+$ 10

——— X ———

Ex./CE/MATH/T/113/2018(S)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester, Supplementary)

Mathematics - II C

Time : Three hours Full Marks : 100

Use a separate Answer Script for each part

Symbols/Notations have their usual meaning.

PART - I

Answer ***q.no. 1*** and any ***three*** questions.

(2 + 3 x 16)

1. Define direction cosines of a straight line.

2. (a) Prove that

$$\left(\vec{A}\times\vec{B}\right).\left(\vec{B}\times\vec{C}\right)\times\left(\vec{C}\times\vec{A}\right)=\left(\vec{A}.\vec{B}\times\vec{C}\right)^2$$

(b) If $\vec{A}=2i+j-3k$ and $\vec{B}=i-2j+k$ find a vector of magnitude 5 and perpendicular to both A and B.

3. (a) Prove that

$$\left(\vec{A}\times\vec{B}\right).\left(\vec{C}\times\vec{D}\right)+\left(\vec{B}\times\vec{C}\right).\left(\vec{A}\times\vec{D}\right)+\left(\vec{C}\times\vec{A}\right).\left(\vec{B}\times\vec{D}\right)=0$$

(b) Prove that law of sines for plane triangle

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

(Turn Over)

4. (a) Find the equation of the image of the line

$$\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{4}$$

in the plane $3x + y - 4z + 21 = 0$

(b) Find the center and the radius of the circle

$$x^2 + y^2 + z^2 = 25,\ \ x+2y+2z+9$$

5. (a) Find the shortest distance between the lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \quad \text{and} \quad \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

(b) The direction cosines of two straight lines are given by equations

$$l - 3m + n = 0 \quad \text{and} \quad l^2 - 5m^2 + n^2 = 0$$

Find the angle between them.

PART - II

Answer any ***five*** questions.

6. (a) Define eigen values of a matrix. Show that if λ is an eigen value of a non singular matrix A, then λ^{-1} is also an eigen value of A^{-1}.

(b) Find the eigen values and the corresponding eigen vector of the matrix

$$A=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$ 4+6

7. (a) Solve the system of equations by Cramer's rule :

$x + 2y - 3z = 1$

$2x - y + z = 4$

$x + 3y = 5$

(b) If $A=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, then show that $A^2 - 4A - 5I_3 = 0$. Hence obtain a matrix B such that $AB = I_3$. 5+5

8. (a) Prove that

$$\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & c^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$$

(b) Solve by matrix method, the system of equations

$x + z = 0$

$3x + 4y + 5z = 2$

$2x + 3y + 4z = 1.$ 4+6

9. (a) If $z = \cos\theta + i\sin\theta$ and n is a tune integer, then show that

$$(1+z)^n + \left(1+\frac{1}{z}\right)^n = 2^{n+1}\cos^n\frac{\theta}{2} + \cos\frac{n\theta}{2}$$