Ex./CE/MATH/T/112/2018

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester)

Mathematics - I C

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer Script for each part.

PART - I

Answer ***q.no. 6*** and any ***three*** from the rest.

1. (a) State Leibnitz theorem on higher order derivatives. Using the theorem find y_n where $y = x^2 \cos x$. 8

   (b) If $y = (x^2 - 1)^n$, prove that

   $$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$ 8

2. (a) State and prove Lagrange's Mean Value theorem. Verify whether the Mean Value theorem is applicable to the function $1 - x^{2/3}$ in the interval $[-1, 2]$. 10

   (b) Evaluate the following limits : (any ***two***) 6

   (i) $\lim_{x \to 0} x^x$

   (ii) $\lim_{x \to 0} \dfrac{\log x}{\operatorname{cosec} x}$

   (iii) $\lim_{x \to 0} \dfrac{5 \sin x - 7 \sin 2x + 3 \sin 3x}{\tan x - x}$

(Turn over)

3. (a) Find the maxima and minima of the function

$$y = \sin x(1 + \cos x) \qquad 4$$

(b) Expand by Maclaurin's theorem upto the terms containing x^4 of the function $y = e^{\sin x}$. 4

(c) Differentiate $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\frac{2x}{1+x^2}$. 4

(d) Find $\frac{dy}{dx}$ in the following case.

$$e^{xy} - x^2 + y^3 = 0 \qquad 4$$

4. (a) Show that the function

$$f(x,y) = \frac{xy}{x^2+y^2},\ (x,y) \neq (0,0)$$

$$= 0,\quad (x,y) = (0,0) \qquad 3$$

(b) Evaluate the following limit

$$\lim_{\substack{x \to 1 \\ y \to 2}} \frac{(x-1)^3}{(x-1)^2 + (y-2)} \qquad 3$$

(c) Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ when $z = x^3 + y^3 + 3axy$. 5

11. (a) Prove that $\int_0^\infty \left(\frac{1}{1+x} - \frac{1}{e^x}\right)^{\frac{1}{x}} dx$ is convergent.

(b) Evaluate $\iint_R (x^2+y^2)\,dxdy$ over R where R is bounded by $y = x^2$, $x = 2$, $y = 1$. 5+5

12. Find the volume of the solid formed by the rotation of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) about the major axis and rotating about ox.

(b) about the minor axis and rotating about oy. 5+5

13. (a) Evaluate $\int_2^3 \frac{dx}{x+1}$ using Simpson's rule with 4 subintervals correct upto 4 decimal places.

(b) Evaluate $\int_0^1 \sqrt{x^2+1}\,dx$ using the Trapezoidal rule with 5 subintervals, correct upto 3 decimal places.

5+5

—— X ——

(d) If $u = \log \frac{x^2 + y^2}{xy}$

verify that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$$ 5

5. (a) State and prove Euler's theorem on homogeneous function. 7

(b) Verify Euler's theorem for

$$u = x^n \sin\left(\frac{y}{x}\right)$$ 4

(c) Find the extremum of the following function :

$u = xy\,(3 - x - y)$ 5

6. Find the derivative of $y = \tan^{-1}(\cos\sqrt{x})$. 2

PART - II

Answer any ***five*** questions.

7. (a) A function f(x) is defined by $f(x) = e^x$ on [a,b]. Find $\int_a^b f(x)dx$ and $\int_{-a}^{b} f(x)dx$. Deduce that f(x) is integrable on [a,b].

(Turn over)

(b) Show that the function f(x) is not integrable on [0,1], where

f(x) = 1, if x rational

= 0, x rational. 5+5

8. (a) Prove that

(i) $\frac{\pi^2}{9} < \int_{\frac{\pi}{6}}^{\pi/2} \frac{x}{\operatorname{Sin} x}\,dx < \frac{2\pi^2}{9}$

(ii) $B(m,m) = 2^{1-2m}\, B\left(m, \frac{1}{2}\right)$ 5+5

9. (a) Find $\int_0^1 \frac{1}{\left(1-x^3\right)^{1/3}}\,dx$

(b) Evaluate $\iint_R xy(x^2+y^2)\,dxdy$

on R : [0,a ; 0.b]. 5+5

10. Examine the convergence of $\int_0^\infty x^{-\frac{1}{2}} e^{-x}\,dx$.

Deduce that $\int_0^1 \frac{1}{\left(1-x^n\right)^{1/n}}\,dx = \frac{\pi}{n} \operatorname{cosec} \frac{\pi}{12}$ n > 1. 5+5