BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2018

(1st Year, 1st Semester, Supplementary)

Mathematics - I C

Time : Three hours Full Marks : 100

Answer any ***ten*** questions.

The notations have their usual meanings.

1. (a) Find y_n for

   (i) $y = \dfrac{1}{x^2 - a^2}$

   (ii) $y = e^{ax} \cos bx$

   (iii) $y = \sin ax$. 6

   (b) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ find $\dfrac{dy}{dx}$. 4

2. (a) State Leibnitz theorem on Successive Differentiaton. Hence find $y = x^3 \log x$. 5

   (b) Given $y = (x^2-1)^n$. Use Leibnitz theorem to show that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$. 5

3. State and prove Rolle's theorem verify Rolle's theorem for

   $f(x) = x^3 - 6x^2 + 11x - 6$ in $[1,3]$. 2+5+3

(Turn Over)

4. State L' Hospital's Rule. Using the rule find the following limits : 10

(i) $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x - \sin x}$

(ii) $\lim\limits_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{\sin x} \right)$

(iii) $\lim\limits_{x \to 0} \log \left(\dfrac{\sin x}{x} \right)^{1/x}$

5. (a) Using Maclaurin's theorem with remainder in Lagrange's form expand sin x in a finite series in powers of x.

(b) State Taylor's theorem (single variable) with Lagrange's form of remainder. 6+4

6. (a) Define the continuity of a function f(x,y) at (a,b).

Examine the continuity of the function given by

$$f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} & , \ where\ x^2 + y^2 \neq 0 \\ 0 & , \ where\ x = 0, y = 0 \end{cases}$$

at the point (0,0). 2+3

12. A river is 80 ft wide. The depth d in feet at distance x ft from one bank is given by the table :

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find the area of cross-section of the river both by

(a) Simpson's $\frac{1}{3}$rd rule and (b) Trapezoidal rule. 10

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(ii) $\int_0^{\frac{\pi}{4}} \tan^6 x \, dx$

(iii) $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta \;\, d\theta$ 6

(b) Find

(i) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$

(ii) $\int_1^{\infty} \frac{dx}{(1+x)^{3/2}}$ 4

10. (a) Prove that B(m,n) = B(n,m). 3

(b) Show that

$$B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}, \; m, \; n>0$$ 7

11. (a) Evaluate $\iiint_R \left(x^2+y^2+z^2\right) dx\,dy\,dz,$ where R is the region bounded by x = 0, y = 0, z = 0 and x + y + z = a, a > 0. 6

(b) Find the moment of intertia of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis. 4

(b) Show that the following limits do not exist :

(i) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{2xy^2}{x^2+y^4}$

(ii) $\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x^2+y^2}$ 5

7. (a) Verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ where $u = \sin^{-1}\frac{x}{y}$. 4

(b) State Euler's theorem on Homogeneous functions of two variables. Verify the theorem for $u = x^n \log\left(\frac{y}{x}\right)$. 6

8. (a) Investigate the maxima and minima for

$$f(x) = x^5 - 5x^4 + 5x^3 - 1$$ 4

(b) A right circular cone with a flat circular base is constructed of sheet material of uniform thickness. Show that for a given volume the area of the surface is a minimum if $\theta = Sin^{-1}\left(\frac{1}{3}\right)$. 6

9. (a) Evaluate

(i) $\int_0^{\pi/2} Cos^2\theta \sin^4\theta \; d\theta$