Ref No. **Ex/Ch.E/T/224/2018(Old)**

B.E. CHEMICAL ENGINEERING 2ND YEAR 2ND SEMESTER EXAMINATION 2018 (Old)

SUBJECT: INTRODUCTION TO TRANSPORT PROCESSES

Time: Three Hours ***Use a separate Answer-Script for each part*** **Full marks 100**

PART I (50 Marks)

Answer any two questions

State all the assumptions; Assume missing data (if any)

No. of questions		Marks
1.	Consider a fluid confined between two stationary parallel plates at a distance δ apart from each other. The upper plate is maintained at a temperature (Tu) greater than that of the lower plate (Tl). Tu>Tl (i) What is the mechanism of heat transport when ΔT=Tu-Tl is very small and fluid is assumed to be stationary? (ii) What is the mechanism of heat transport for large value of ΔT? (iii) What is the mechanism of heat transport when upper plate is moving at a velocity U instead of being stationary?	(1+2+2)
	Answer either 2(a) or 2(b)	
2.(a)	A large deep lake, which initially had a uniform oxygen (A) concentration of C_{Ai} (kg/m^3), has its surface concentration suddenly raised and maintained at C_{As} (kg/m^3) concentration level. Derive the governing equation for the transfer of oxygen into the lake with simultaneous disappearance of oxygen by a first order biological reaction. Write the boundary conditions. (DO NOT SOLVE)	
2(b)	Consider the flow of a Newtonian, incompressible fluid of constant properties through a circular pipe of radius R and length L. The driving force of the flow, the pressure difference is a function of time (example cardio-vascular flow): $p=p_o+\varepsilon\sin(kt)$. Derive the governing equation that must be solved to obtain the velocity distribution inside the pipe and write the boundary conditions. (DO NOT SOLVE)	(10)

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SUBJECT: INTRODUCTION TO TRANSPORT PROCESSES
Time: Three Hours **Full marks 100**

No. of question		Marks
	Answer either 3(a) or 3(b)	
3(a)	Consider the steady laminar flow of an incompressible isothermal Newtonian fluid filled between two coaxial cylinders of radii R(outer) and kR(inner) rotating at angular velocities Ωo and Ωi, respectively. Derive the angular velocity profile of the fluid ($kR \le r \le R$).	
3(b)	An incompressible, Newtonian fluid is flowing downward (along z) like a film of thickness δ, along the outside wall of a circular pipe of radius R and length L under gravitational force. The film is subjected to atmospheric pressure (at the top and the bottom). Derive the velocity distribution $v_Z(r)$ of the fluid. Continuity and Navier stokes equation for cylindrical coordinate are given below	(12)
	$\frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial v_z}{\partial z}=0$	
	$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^{\ 2}}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho g_r$	
	$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\rho g_\theta$	
	$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}++\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$	

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SUBJECT: INTRODUCTION TO TRANSPORT PROCESSES

Time: Three Hours **Full marks 100**

No. of questions		Marks
	Answer either 4(a) or 4(b)	
4(a)	Consider the process of oxygen transfer from the interior lung cavity, across the lung tissue, to the network of blood vessels on the opposite side. The lung tissue (species B) may be approximated as a plane wall of thickness L. The inhalation process may be assumed to maintain a constant molar concentration C_{Ao} of oxygen (Species A) in the tissue at its inner surface (x=0), and assimilation of oxygen by the blood may be assumed to maintain a constant molar concentration C_{AL} of oxygen in the tissue at its outer surface (x=L). There is oxygen consumption in the tissue due to metabolic processes, and the reaction is zero order with $r_A''' = -k_o$. Obtain expressions for (i) the distribution of oxygen concentration in the tissue (ii) the rate of assimilation of oxygen by the blood per unit tissue surface area. [Show the derivation of governing equation and solution]	
4(b)	Asphalt pavement may achieve temperatures as high as 50^{0}C on a hot summer day. Assume that such a temperature exists throughout the pavement, when suddenly a rainstorm reduces the surface temperature to 20^{0}C. Calculate the total amount of energy (J/m^2) that will be transferred from the asphalt over a 30-min period in which the surface is maintained at 20^{0}C. [Show the derivation of governing equation and solution]	**(15)**
5(a)	Consider the thermal boundary layer over a flat surface. Define heat transfer coefficient and derive an expression for the non-dimensional temperature gradient ($\frac{\partial T^*}{\partial y^*}$) at the solid fluid interface (y*= 0).	**(1+2)**
5(b)	Experimental tests on a portion of turbine blade having characteristic lengthscale (chord length) 40 mm, indicate a heat flux to the blade of q''=95,000 W/m^2. The blade operates in an airflow at Tα=1150°C and V=160 m/s. Steady state surface temperature of 800°C,is maintained by circulating a coolant inside the blade. (a) Calculate the heat transfer coefficient. (b) Determine the heat flux at the same dimensionless location of a geometrically similar turbine blade having a chord length of L=80 mm, when the blade operates in an airflow at Tα=1150°C and V=80 m/s, with Ts=800°C.	**(2+3)**

Error Function Table

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

	Hundredths digit of x									
x	0	1	2	3	4	5	6	7	8	9
0.0	0.00000	0.01128	0.02256	0.03384	0.04511	0.05637	0.06762	0.07886	0.09008	0.10128
0.1	0.11246	0.12362	0.13476	0.14587	0.15695	0.16800	0.17901	0.18999	0.20094	0.21184
0.2	0.22270	0.23352	0.24430	0.25502	0.26570	0.27633	0.28690	0.29742	0.30788	0.31828
0.3	0.32863	0.33891	0.34913	0.35928	0.36936	0.37938	0.38933	0.39921	0.40901	0.41874
0.4	0.42839	0.43797	0.44747	0.45689	0.46623	0.47548	0.48466	0.49375	0.50275	0.51167
0.5	0.52050	0.52924	0.53790	0.54646	0.55494	0.56332	0.57162	0.57982	0.58792	0.59594
0.6	0.60386	0.61168	0.61941	0.62705	0.63459	0.64203	0.64938	0.65663	0.66378	0.67084
0.7	0.67780	0.68467	0.69143	0.69810	0.70468	0.71116	0.71754	0.72382	0.73001	0.73610
0.8	0.74210	0.74800	0.75381	0.75952	0.76514	0.77067	0.77610	0.78144	0.78669	0.79184
0.9	0.79691	0.80188	0.80677	0.81156	0.81627	0.82089	0.82542	0.82987	0.83423	0.83851
1.0	0.84270	0.84681	0.85084	0.85478	0.85865	0.86244	0.86614	0.86977	0.87333	0.87680
1.1	0.88021	0.88353	0.88679	0.88997	0.89308	0.89612	0.89910	0.90200	0.90484	0.90761
1.2	0.91031	0.91296	0.91553	0.91805	0.92051	0.92290	0.92524	0.92751	0.92973	0.93190
1.3	0.93401	0.93606	0.93807	0.94002	0.94191	0.94376	0.94556	0.94731	0.94902	0.95067
1.4	0.95229	0.95385	0.95538	0.95686	0.95830	0.95970	0.96105	0.96237	0.96365	0.96490
1.5	0.96611	0.96728	0.96841	0.96952	0.97059	0.97162	0.97263	0.97360	0.97455	0.97546
1.6	0.97635	0.97721	0.97804	0.97884	0.97962	0.98038	0.98110	0.98181	0.98249	0.98315
1.7	0.98379	0.98441	0.98500	0.98558	0.98613	0.98667	0.98719	0.98769	0.98817	0.98864
1.8	0.98909	0.98952	0.98994	0.99035	0.99074	0.99111	0.99147	0.99182	0.99216	0.99248
1.9	0.99279	0.99309	0.99338	0.99366	0.99392	0.99418	0.99443	0.99466	0.99489	0.99511
2.0	0.99532	0.99552	0.99572	0.99591	0.99609	0.99626	0.99642	0.99658	0.99673	0.99688
2.1	0.99702	0.99715	0.99728	0.99741	0.99753	0.99764	0.99775	0.99785	0.99795	0.99805
2.2	0.99814	0.99822	0.99831	0.99839	0.99846	0.99854	0.99861	0.99867	0.99874	0.99880
2.3	0.99886	0.99891	0.99897	0.99902	0.99906	0.99911	0.99915	0.99920	0.99924	0.99928
2.4	0.99931	0.99935	0.99938	0.99941	0.99944	0.99947	0.99950	0.99952	0.99955	0.99957
2.5	0.99959	0.99961	0.99963	0.99965	0.99967	0.99969	0.99971	0.99972	0.99974	0.99975
2.6	0.99976	0.99978	0.99979	0.99980	0.99981	0.99982	0.99983	0.99984	0.99985	0.99986
2.7	0.99987	0.99987	0.99988	0.99989	0.99989	0.99990	0.99991	0.99991	0.99992	0.99992
2.8	0.99992	0.99993	0.99993	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	0.99996
2.9	0.99996	0.99996	0.99996	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998
3.0	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999	0.99999
3.1	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
3.2	0.99999	0.99999	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Complementary Error Function Table

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2}\,dt$$

	Hundredths digit of x									
x	0	1	2	3	4	5	6	7	8	9
0.0	1.00000	0.98872	0.97744	0.96616	0.95489	0.94363	0.93238	0.92114	0.90992	0.89872
0.1	0.88754	0.87638	0.86524	0.85413	0.84305	0.83200	0.82099	0.81001	0.79906	0.78816
0.2	0.77730	0.76648	0.75570	0.74498	0.73430	0.72367	0.71310	0.70258	0.69212	0.68172
0.3	0.67137	0.66109	0.65087	0.64072	0.63064	0.62062	0.61067	0.60079	0.59099	0.58126
0.4	0.57161	0.56203	0.55253	0.54311	0.53377	0.52452	0.51534	0.50625	0.49725	0.48833
0.5	0.47950	0.47076	0.46210	0.45354	0.44506	0.43668	0.42838	0.42018	0.41208	0.40406
0.6	0.39614	0.38832	0.38059	0.37295	0.36541	0.35797	0.35062	0.34337	0.33622	0.32916
0.7	0.32220	0.31533	0.30857	0.30190	0.29532	0.28884	0.28246	0.27618	0.26999	0.26390
0.8	0.25790	0.25200	0.24619	0.24048	0.23486	0.22933	0.22390	0.21856	0.21331	0.20816
0.9	0.20309	0.19812	0.19323	0.18844	0.18373	0.17911	0.17458	0.17013	0.16577	0.16149
1.0	0.15730	0.15319	0.14916	0.14522	0.14135	0.13756	0.13386	0.13023	0.12667	0.12320
1.1	0.11979	0.11647	0.11321	0.11003	0.10692	0.10388	0.10090	0.09800	0.09516	0.09239
1.2	0.08969	0.08704	0.08447	0.08195	0.07949	0.07710	0.07476	0.07249	0.07027	0.06810
1.3	0.06599	0.06394	0.06193	0.05998	0.05809	0.05624	0.05444	0.05269	0.05098	0.04933
1.4	0.04771	0.04615	0.04462	0.04314	0.04170	0.04030	0.03895	0.03763	0.03635	0.03510
1.5	0.03389	0.03272	0.03159	0.03048	0.02941	0.02838	0.02737	0.02640	0.02545	0.02454
1.6	0.02365	0.02279	0.02196	0.02116	0.02038	0.01962	0.01890	0.01819	0.01751	0.01685
1.7	0.01621	0.01559	0.01500	0.01442	0.01387	0.01333	0.01281	0.01231	0.01183	0.01136
1.8	0.01091	0.01048	0.01006	0.00965	0.00926	0.00889	0.00853	0.00818	0.00784	0.00752
1.9	0.00721	0.00691	0.00662	0.00634	0.00608	0.00582	0.00557	0.00534	0.00511	0.00489
2.0	0.00468	0.00448	0.00428	0.00409	0.00391	0.00374	0.00358	0.00342	0.00327	0.00312
2.1	0.00298	0.00285	0.00272	0.00259	0.00247	0.00236	0.00225	0.00215	0.00205	0.00195
2.2	0.00186	0.00178	0.00169	0.00161	0.00154	0.00146	0.00139	0.00133	0.00126	0.00120
2.3	0.00114	0.00109	0.00103	0.00098	0.00094	0.00089	0.00085	0.00080	0.00076	0.00072
2.4	0.00069	0.00065	0.00062	0.00059	0.00056	0.00053	0.00050	0.00048	0.00045	0.00043
2.5	0.00041	0.00039	0.00037	0.00035	0.00033	0.00031	0.00029	0.00028	0.00026	0.00025
2.6	0.00024	0.00022	0.00021	0.00020	0.00019	0.00018	0.00017	0.00016	0.00015	0.00014
2.7	0.00013	0.00013	0.00012	0.00011	0.00011	0.00010	0.00009	0.00009	0.00008	0.00008
2.8	0.00008	0.00007	0.00007	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005	0.00004
2.9	0.00004	0.00004	0.00004	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002
3.0	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002	0.00001	0.00001	0.00001
3.1	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
3.2	0.00001	0.00001	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

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Ex/ChE/T/224/2018 (Old)

B.E. CHEMICAL ENGINEERING SECOND YEAR SECOND SEMESTER (Old) - 2018

INTRODUCTION TO TRANSPORT PROCESSES

Time: Three Hours

Full Marks : 100

Part II (50 Marks)

Answer any five questions.
All the questions carry equal marks.

1. Using Gauss divergence theorem of vector calculus, derive the differential equation of energy conservation, assuming the volume of the system to be fixed without any work being transferred and its mass remaining constant.
2. Derive an expression for the species continuity equation considering a differential control volume for species conservation in two-dimensional flow of a viscous fluid with mass transfer.
3. Prove that the flow of a liquid in laminar flow between two infinite parallel flat plates is given by $P_0 - P_L = \frac{12\mu v_{avg} L}{a^2}$
   where L is length of plate in the direction of flow, a is the distance between plates.
4. Derive the concentration profile in a spherical gas film for diffusion with heterogeneous chemical reaction.
5. A test on water in a capillary viscometer gave the following data: Flow rate = 880 mm^3/sec, Tube length = 1 m, Tube diameter = 0.5 mm, Pressure drop = 10 mPa. Determine the viscosity of water and test that the flow is in laminar region.
6. An oil is acting as a lubricant in between a pair of cylindrical surfaces. The angular velocity of the outer cylinder is 7900 rpm. Outer cylinder has a radius of 6 cm and the clearance between the cylinders is 0.02 cm. What is the maximum temperature of the oil if both wall temperatures are at 160°C? The physical properties of oil are $\mu = 92.3 \times 10^{-3}$ N.s/m^2; $\rho = 1200$ kg/m^3 and $k =$ 2.5 W/m.°C.
7. Consider the flow of an incompressible Newtonian fluid with uniform-viscosity, ***without*** body forces. Non-dimensionalize the governing equations for energy balance in such a way that the non-dimensional form contains two or more of the following dimensionless numbers: *Reynolds Number, Prandtl Number and Eckert Number*. What is the physical significance of *Eckert Number*.

Continuity Equation

Cylindrical

$$\frac{\partial \rho}{\partial t} + \left\{\frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial(\rho V_z)}{\partial z}\right\} = 0$$

Navier Stokes Equations

Cartesian

$$x: \quad \rho\left\{\frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z}\right\} = \rho g_x - \frac{\partial p}{\partial x} + \mu\left\{\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}\right\}$$

$$y: \quad \rho\left\{\frac{\partial V_y}{\partial t} + V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z}\right\} = \rho g_y - \frac{\partial p}{\partial y} + \mu\left\{\frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} + \frac{\partial^2 V_y}{\partial z^2}\right\}$$

$$z: \quad \rho\left\{\frac{\partial V_z}{\partial t} + V_x\frac{\partial V_z}{\partial x} + V_y\frac{\partial V_z}{\partial y} + V_z\frac{\partial V_z}{\partial z}\right\} = \rho g_z - \frac{\partial p}{\partial z} + \mu\left\{\frac{\partial^2 V_z}{\partial x^2} + \frac{\partial^2 V_z}{\partial y^2} + \frac{\partial^2 V_z}{\partial z^2}\right\}$$

Cylindrical

r:

$$\rho\left\{\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z}\right\}$$
$$= \rho g_r - \frac{\partial \rho}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)\right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial^2 V_r}{\partial z^2}\right\}$$

θ:

$$\rho\left\{\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z}\right\}$$
$$= \rho g_\theta - \frac{1}{r}\frac{\partial \rho}{\partial \theta} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} + \frac{\partial^2 V_\theta}{\partial z^2}\right\}$$

z:

$$\rho\left\{\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right\} = \rho g_z - \frac{\partial \rho}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right\}$$

Mass Diffusion Equation

Spherical Coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(D_{AB}r^2\frac{\partial C_A}{\partial r}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial \emptyset}\left(D_{AB}\frac{\partial C_A}{\partial \emptyset}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(D_{AB}sin\theta\frac{\partial C_A}{\partial \theta}\right) + \dot{N_A} = \frac{\partial C_A}{\partial t}$$

Equation of Energy for Pure Newtonian Fluid with constant ρ and k

$$\frac{\delta T}{\delta t} + v_r\frac{\delta T}{\delta r} + \frac{v_\theta}{r}\frac{\delta T}{\delta \theta} + v_z\frac{\delta T}{\delta z} = \frac{k}{\hat{\rho} c_p}\left[\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta T}{\delta r}\right) + \frac{1}{r^2}\left(\frac{\delta^2 T}{\delta \theta^2}\right) + \frac{\delta^2 T}{\delta z^2}\right] + \frac{\mu \emptyset_v}{\hat{\rho} c_p} + \frac{S_C}{\hat{\rho} c_p}$$

[Turn over

Dissipation Function for Newtonian Fluids

$$\o_v = 2\left(\left(\frac{\delta v_r}{\delta r}\right)^2 + \left(\frac{1}{r}\frac{\delta v_\theta}{\delta\theta} + \frac{v_r}{r}\right)^2 + \left(\frac{\delta v_z}{z}\right)^2\right) + \left[r\frac{\delta}{\delta r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\delta v_r}{\delta\theta}\right]^2 + [\frac{1}{r}\frac{\delta v_z}{\delta\theta} + \frac{\delta v_\theta}{\delta z}]^2 + \left[\frac{\delta v_r}{\delta z} + \frac{\delta v_z}{\delta r}\right]^2 - \frac{2}{3}\left[\frac{1}{r}\frac{\delta}{\delta r}(rv_r) + \frac{1}{r}\frac{\delta v_\theta}{\delta\theta} + \frac{\delta v_z}{\delta z}\right]^2$$