13. a) Classify the isolated singularities of the following functions :

i) $f(z) = \dfrac{z}{(z+1)(z-2)}$

ii) $f(z) = \dfrac{\operatorname{Sin} z}{e^z - 1}$ $\quad 2\frac{1}{2} \times 2$

b) Show that the integral $\displaystyle\int_0^\infty \frac{dx}{x^4 + a^4} = \frac{\pi}{2\sqrt{2}a^3}$ using residues.

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Ex/CHE/Math/T/226/2018 (Old)

BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2018

(2nd Year, 2nd Semester)

MATHEMATICS - IVB

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I (50 MARKS)

Answer any ***five*** questions

1. a) Define the equation of the tangent plane at a given point on a surface. Hence find the equation of the tangent plane at the point (3, –2, 1) to the surface $xy^2 + 2yz = 8$.

b) Show that the vector field $\vec{v} = xy(2yz\hat{i} + 2xz\hat{j} + xy\hat{k})$ is conservative. 5+5

2. a) Evaluate the line integral $\int_C \vec{v} \cdot d\vec{r}$, where $\vec{v} = x^2\hat{i} - 2y^3\hat{j} + z\hat{k}$ and C is the straight line path joining (–1, 2, 3) to (2, 3, 5).

b) Evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n}\, dA$, where $\vec{F} = 6z\hat{i} + 6\hat{j} + 3y\hat{k}$ and S is the projection of the plane $2x + 3y + 4z = 12$, which is in the first octant. State Green's theorem. 5+5

[Turn over

3. State Gauss's divergence theorem. Verify the theorem for $\vec{A} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $y = x, y = x^2$. 10

4. a) Using Green's theorem evaluate $\oint (xy + y^2)dx + x^2dy$ around the boundary of the region defined by $y = x, y = x^2$.

   b) Show that $\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r} \cdot \vec{n}}{r^2} ds$, where S is a closed surface. 6+4=10

5. a) If f(t) is of exponential order γ as $t \to \infty$ and is piecewise continuous over every finite interval of $t \geq 0$, then show that Laplace transform of f(t) exits for $S > \gamma$, where S is Laplace transform variable. 4

   b) Find inverse Laplace transform of

   $$\frac{3S+1}{S^2(S^2+4)} e^{-3S}$$ 3

   c) Solve by Laplace transform method

   $$t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = \cos t, \text{ given that } y(0) = 1.$$ 3

10. a) Prove that the Cauchy-Riemann conditions are necessary for differentiability of a complex function f(z) at a point z_o.

    b) Evaluate $\int_C \frac{z^3 + 5z - 7}{\text{Cos}z} dz$, where C denotes the rectangle with vertices $\pm\frac{\pi_i}{8}, \frac{\pi(1 \pm i)}{8}$. 5×2=10

11. a) Use Cauchy's integral formula to evaluate the integral $\int_C \frac{e^z}{z^2(z+1)^3} dz$, where C is the circle $|z| = 2$ in the counterclockwise direction.

    b) Obtain the terms up to z^2 in the Taylor series expansion of $f(z) = \frac{z^2 + \sin^2 z}{1 - \cos z}$ about the point $z = 0$. What is its radius of convergence ? 5+5

12. a) Find the Laurent expansion of the function $f(z) = \frac{z^3}{z-1}$ about the point 1.

    b) Let $f(z) = \frac{g(z)}{(z-a) \cdot h(z)}$, where g and h are analytic functions with $h(a) \neq 0$. Prove that the residue of the function f at $z = a$ is $\frac{g(a)}{h'(a)}$. 5+5

[Turn over

c) Define Christoffel symbol of the 1st kind and 2nd kind. If $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta(d\phi)^2$, find the values of

i) [22, 1] and [13, 3]   ii) $\left\{\begin{matrix} 1 \\ 22 \end{matrix}\right\}$ and $\left\{\begin{matrix} 3 \\ 13 \end{matrix}\right\}$ 4

PART - II (50 MARKS)

Answer Questions ***8*** and any ***three*** from the rest.

5×4=20

8. a) Find all values of z such that Sinz = 2.

b) If $f(z)$ is a continuous function on $\mathbb{C}$, what can you say about the continuity of $\overline{f(\overline{z})}$? Justify your answer.

c) Suppose that f(z) is a complex analytic function with the property $f'(z) = f(z)$, for all $z \in \mathbb{C}$. Prove that $f(z) = ke^z$, for some $k \in \mathbb{C}$.

9. a) Find $f'(z)$, where

$$f(Z) = (r^2\cos 2\theta + r\cos\theta) + i(r^2\mathrm{Sin}2\theta + r\mathrm{Sin}\theta).$$

b) Let $u(x, y) = y^3 - 3x^2y$. Find the corresponding conjugate harmonic function v(x, y) and construct the analytic function $f(Z) = u(x, y) + iv(x, y)$.   5×2=10

6. a) Find Fourier transform of $f(x)$ defined by

$$f(x) = \begin{cases} 1, & |x| \le a \\ 0, & |x| > a \end{cases}$$

and hence evaluate $\int_0^\infty \frac{\sin x}{x}dx.$ 4

b) Find the inverse Fourier transform of

$$F(S) = \frac{1}{(S^2+4)(S^2+9)}$$ 3

c) Find z-transform of

i) $e^1\sin 2t$ ii) $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$ 3

7. a) Define covariant and contravariant tensors of order two. A covariant tensor has components xy, $2y - z^2$, xz in rectangular co-ordinates. Find its convariant components in spherical co-ordinates.   4

b) Show that the kronecker delta is a mixed tensor of order two.   2