11. a) Solve the partial differential equation

$$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u,\ \ u(0,y)=e^{-5y}$$

by the method of separation of variables.

b) Solve $y^2p-xyq=x(z-2y)$ where p, q have usual meanings. 6+4

12. a) Express $J_5(x)$ in terms of $J_o(x)$ and $J_1(x)$.

b) Solve

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$$ 5+5

Ex/CHE/Math/T/215/2018

Bachelor of Engineering in Chemical Engineering Examination, 2018

(2nd Year, 1st Semester)

Mathematics - III

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use separate answer script for each part.

(Unexplained Notations and Symbols have their usual meanings)

PART - I (50 marks)

Answer ***Q.No. 6*** and any ***three*** from the rest.

1. a) Suppose V is an inner product space. Define

i) Orthogonality of two vectors *v*, w of v

ii) Orthogonality of a subset S of V,

iii) Orthonormality of a subset S of V

Give example in each case. 3

b) Let $S=\{\nu_1,\cdots,\nu_k\}$ be an orthogonal set in an inner product space V. What do you mean by (i) Fourier expansion, (ii) Fourier co-efficients of a vector $\nu\in V$ with respect to S ? 2

c) Verify that (i) $\{1, \cos nx, \sin nx\}_{n=1}^{\infty}$ is an orthogonal set in $C[-\pi,\pi]$ with usual inner product,

[Turn over

(ii) $\left\{1, \cos\frac{n\pi x}{l}, \sin\frac{n\pi x}{l}\right\}_{n=1}^{\infty}$

is an orthogonal set in $C[-l, l]$ with usual inner product. 5

d) Determine the Fourier co-efficients of a function f in each of the following cases :

i) $f \in C[-\pi, \pi]$ and the orthogonal set is

$\{1. \text{Cos}\, nx. \sin nx\}_{n=1}^{\infty}$,

ii) $f \in C[-l, l]\ (l > 0)$ any real number and the orthogonal set is

$\left\{1, \text{Cos}\frac{n\pi x}{l}, \frac{\text{Sin}\ n\pi x}{l}\right\}_{n=1}^{\infty}$ 6

2. a) Let f be a real vlaued function defined on $[0, \pi]$. Define (i) the even extension f_e and (ii) the odd extension f_o of f to $(-\pi, \pi)$. 2

b) Prove that f_e is an even function and f_o is an odd function. 2

c) Prove that for a function $f \in C[0, \pi]$ the Fourier sine series is equal to the fourier series of f_o on $[-\pi, \pi]$. 4

d) Find the Fourier cosine series of $f(x) = \sin x$ on $[0, \pi]$. 4

PART - II

Answer ***any five*** questions.

7. Solve

a) $(xy\sin xy + \cos xy)ydx + (xy\sin xy - \cos xy)xdy = 0$

b) $y_2 + 9y = \sec 3x$ by the method of variation of parameters. 5+5

8. Solve

a) $D^2(D+1)^2(D^2 + D + 1)^2 y = e^x$

b) $(r + \sin\theta - \cos\theta)dr + r(\sin\theta + \cos\theta)d\theta = 0$ 6+4

9. a) Find the solution in sereis of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2 y = 0$ about at $x = 0$.

b) Form the partial differential equation from $f(x^2 + y^2, z - xy) = 0$ 5+5

10. a) Express $2 - 3x + 4x^2$ in terms of Legendre polynomials.

b) Prove that

$$1 + \frac{1}{2}P_1(\cos\theta) + \frac{1}{3}P_2(\cos\theta) + \cdots + \cdots = \log\left(\frac{1 + \sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)$$

4+6

b) the system

$$\frac{dx}{dt} - 6x + 3y = 8e^{t}$$

$$\frac{dy}{dt} - 2x - y = 4e^{t}$$

satisfying $x(0) = -1,\ y(0) = 0$ 9+7

5. a) Find the Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} dx$ 8

b) Using convolution theorem, evaluate the inverse Z-transform of $\frac{z^2}{(z-a)(z-b)}$ 6

c) Find the Z-transform of $e^t \sin 2t$. 2

6. Evaluate $L^{-1}\left\{\frac{3}{s}(e^{-3s} - e^{-5s})\right\}$. 2

e) From the Fourier series of $f(x) = x$ using parseval's equality deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. 4

3. a) Find $L\{f(t)\}$ if $f''(t) + 3f'(t) + 2f(t) = 0,\ f(0) = 1,$ $f'(0) = 2.$ 2

b) Evaluate the integral $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t} dt$ using laplace transform, where a, b are positive real numbers. 3

c) Evaluate

i) $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$

ii) $L^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\}$ 4+4

d) Using convolution theorem evaluate

$$L^{-1}\left\{\frac{1}{(s+1)(s+9)^2}\right\}$$ 3

4. Using Laplace transform method solve

a) $\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = 10\cos t,$

$y(0) = 0,\ y'(0) = 0,\ y''(0) = 3$

[Turn over