8. (a) Solve : $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$

(b) Find integrating factor of the equation $(x^4y^2 - y)dx + (x^2y^4 - x)dy = 0$ and hence solve it. 8+7

9. (a) A lightly stretched string with fixed end points x=0 and x=e is initially in a position given by $y(x,0) = y_0 \sin^3\left(\dfrac{\pi x}{e}\right)$. If it is released from rest from this position, find the displacement y at any time t.

(b) A rod of length e with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0°C and kept at that temperature. Find the temperature function u(x,t). 8+7

10. (a) Let $P_n(x)$ denote the Legendre polynomial of degree n. Prove that

$$x^4 = \frac{1}{5} P_0(x) + \frac{4}{7} P_2(x) + \frac{8}{35} P_4(x)$$

(b) Find the complete integral of the PDE $z^2 = pqxy$. 7+8

——— X ———

Ex./CHE/MATH/T/216/2018(S)(OLD)

BACHELOR OF ENGINEERING CHEMICAL EXAMINATION, 2018

(2nd Year, 1st Semester, Supplementary, Old Syllabus)

Mathematics - III B

Time : Three hours Full Marks : 100

Use a separate Answer-Script for each part.

PART - I (40 marks)

Answer ***q.no. 5*** and any ***three*** from the rest.

Symbols/Notations have their usual meaning.

1. (a) Suppose f is a Riemann integrable function on $[-\pi, \pi]$. What is meant by the Fourier series of f on $[-\pi, \pi]$?

(b) Find the Fourier series of f defined by

$f(x) = 0 , -\pi \le x \le 0$

$= \sin x \quad 0 \le x \le \pi.$

Hence deduce that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + ... = \frac{1}{2}$$ 2+(8+2)

2. Obtain the Fourier series for

$$f(x) = \begin{cases} x\ , & -1 < x \le 0 \\ x+2, & 0 < x < 1 \end{cases}$$

Hence deduce the sum of

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$$ 12

3. Discuss the convergence of the series

(i) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{2n^3+1}}$ (ii) $\sum_{n=1}^{\infty} \left(\sqrt[3]{1+n^3} - n\right)$ 4+8

4. (a) Prove that if the series $\sum_{n=1}^{\infty} a_n$ converges then $\lim_{n\to\infty} a_n = 0$. Give an example to illustrate that the converse is not true.

(b) State and prove alternating series test. 6+6

5. What do we usually mean by

(a) Fourier sine series,

(b) Fourier cosine series of a Riemann integrable function over $[0, \pi]$? Find Fourier co-efficients in each case. 4

PART - II (60 marks)

Answer any ***four*** questions.

6. (a) Solve the equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0$ near the ordinary point $x = 0$.

(b) Show that Bessel functions of first kind of order zero can be put in the form

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n\,!)^2} \left(\frac{x}{2}\right)^{2n}$$ 8+7

7. (a) Form the PDE by eliminating the arbitrary functions from

(i) $z = f\left(\frac{xy}{z}\right)$ (ii) $f(x+y+z, x^2+y^2+z^2) = 0$

(b) Find the general integrals of the following linear PDE

$$(x^2 - yz)p + (y^2 - zx)\,z = z^2 - xy$$ (4+5)+6