Ex./CHE/MATH/T/215/2018(S)

BACHELOR OF ENGINEERING CHEMICAL EXAMINATION, 2018

(2nd Year, 1st Semester, Supplementary)

Mathematics - III

Time : Three hours Full Marks : 100

Use a separate Answer-Script for each part.

PART - I (40 marks)

Answer ***q.no. 5*** and any ***three*** from the rest.

Symbols/Notations have their usual meaning.

1. (a) Suppose f is a Riemann integrable function on $[-\pi,\pi]$. What is meant by the Fourier series of f on $[-\pi,\pi]$?

   (b) Find the Fourier series of f defined by

   $$f(x) = 0 ,\ -\pi \le x \le 0$$
   $$= \sin x,\ 0 \le x \le \pi.$$

   Hence deduce that

   $$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + ... = \frac{1}{2}$$ 2+(8+2)

2. (a) Find the laplace transform of

   $$2^t + \frac{\cos 2t + \cos 3t}{t} + t\sin t$$

(Turn Over)

(b) (i) Find the inverse Laplace transform of

$$\frac{1}{\left(s^2+a^2\right)^z} \ (a \text{ is } a \text{ constant})$$

(ii) Using convolution theorem find the inverse Laplace transform of $\dfrac{s}{\left(s^2+1\right)\left(s^2+4\right)}$. 4+(4+4)

3. (a) Use the method of Laplace transform to solve

(i) $t\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + ty = \cos t$ given that $y(0)=1$.

(ii) $\dfrac{d^2y}{dt^2} + 9y = \cos 2t$ given that $y(0)=1$, $y\left(\dfrac{\pi}{2}\right)=1$.

(b) Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & for \quad 0<x<1 \\ 2-x & for \quad 1<x<2 \\ 0 & for \quad x>2 \end{cases}$$

(4+4)+4

4. (a) $P_0(x)=1$, $P_1(x)=x$ and $P_2(x)=\dfrac{3}{2}x^2-\dfrac{1}{2}$ are orthogonal on [–1,1] with respect to the usual inner product.

(b) Find the solution of the equation

$\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 4x = 3\sin 2t$ which will satisfy $x=0$, $\dfrac{dx}{dt}=0$ at $t=0$.

(c) Solve by the method variation of parameters the equation $\dfrac{d^2y}{dx^2} + 9y = \sec 3x$. 5+5+5

11. (a) Solve $(e^x+1)y\,dy = (y^2+1)e^x\,dx$, given $y = 0$ when $x = 0$.

(b) Show that all circles of radius r are represented by the differential equation

$$\left(1+\left(\frac{dy}{dx}\right)^2\right)^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$$

(c) Solve the equation $(y^2 e^x + 2xy)dx - x^2\,dy = 0$.

5+5+5

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7. (a) Form the PDE by eliminating the arbitrary functions from

(i) $z = f\left(\frac{xy}{z}\right)$ (ii) $f(x+y+z, x^2+y^2+z^2)=0$

(b) Find the general integrals of the following linear PDE

$(x^2 - yz)p + (y^2 - zx)\,q = z^2 - xy$ (4+5)+6

8. (a) Solve : $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(b) Find an integrating factor of the equation $(x^4y^2 - y)dx + (x^2y^4 - x)dy = 0$ and hence solve it. 8+7

9. (a) Find the complete integral of the PDE $z^2 = pqxy$.

(b) Find the solution of the two dimensional Laplace equation by the method of separation of variables. 6+9

10. (a) Let $P_n(x)$ denote the Legendre polynomial of degree n. Prove that

$$x^4 = \frac{1}{5} P_0(x) + \frac{4}{7} P_2(x) + \frac{8}{35} P_4(x)$$

Find the constant C_0, C_1, C_2 such that $C_0 P_0(x) + C_1 P_1(x) + C_2 P_2(x)$ is the Fourier expansion of f on $[-1,1]$.

(b) Find the inverse z-transform of

$$\frac{2z^2 + 3z}{(z+2)(z-4)}$$ 6+6

5. What do we usually mean by

(i) Fourier sine series,

(ii) Fourier cosine series of a Riemann integrable function over $[0, \pi]$? Find Fourier co-efficients in each case. 4

PART - II (60 marks)

Answer any ***four*** questions.

6. (a) Solve the equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = 0$ near the ordinary point $x=0$.

(b) Show that Bessel functions of first kind of order zero can be put in the form

$$J_0(x) = \sum_{x=0}^{\infty} \frac{(-1)^n}{(n\,!)^2} \left(\frac{x}{2}\right)^{2n}$$ 8+7